August 2020

A Holistic Approach to Developmental Mathematics

Leah M. Rineck
University of Wisconsin-Milwaukee

A HOLISTIC DEVELOPMENTAL MATHEMATICS COURSE FOR ALL LEARNERS

by

Leah M. Rineck

A Dissertation Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

in Urban Education

at

The University of Wisconsin – Milwaukee

August 2020

ABSTRACT

A HOLISTIC DEVELOPMENTAL MATHEMATICS COURSE FOR ALL LEARNERS

by

Leah M. Rineck

The University of Wisconsin – Milwaukee, 2020
Under the Supervision of Professor Michael Steele

The purpose of this study was to develop a holistic course that meets the needs of students referred to multiple developmental mathematics courses. The study aimed to determine if the holistic design contributed to an increase in conceptual understanding of fractions, integers, and their operations. Additionally, the study analyzed the pass rates from the course design and compared these pass rates to historic versions of the course and current gateway courses. Last, the student's perceptions of the course were analyzed to determine their perceptions of the design features of the course. The study took place at a large midwestern urban university. This university is virtually an open access university. The students come from diverse backgrounds and were referred to multiple levels of developmental mathematics by a placement exam or their ACT Math score. The course discusses material from basic math through beginning algebra. The study was done using a design-experiment based on four design principles. First, create an equitable environment for all students. Second, provide learning activities that promote a conceptual understanding of mathematics. Third, provide growth mindset and study skills instruction. Fourth, develop a grading system that emphasizes student learning. The results of the study indicate that the course contributed to an increase in conceptual understanding of fractions, integers and their operations on all learning outcomes that were assessed. Many of the learning outcomes increased to show proficiency. Additionally, the pass rate for the course was 80% and there was no statistically significant difference in pass rate based on a student's race. Furthermore, students that continued to their gateway mathematics course passed that course at the same rate or better than students that started in the gateway course. Last, students perceived that the design features that were most beneficial to their learning were the ones that directly applied to their math instruction. This course provides a holistic approach towards developmental mathematics instruction that addresses the needs of students when they walk in the door. Students are given the tools they need to be successful in this course and any other math courses they need to take.

To

my mom,

without you none of this would be possible

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

Who knew a quiet dinner five years ago could amount to a doctorate? This whole endeavor would not have been possible without the unending support from my husband, Mark. He was there through thick and thin. He has even tried to understand my research. I never would have been able to complete this work without him by my side. Thank you!

Next, I want to thank my daughters, Kayley and Natalie. Kayley, I have no idea how you knew the exact minute to tell me you were proud of me. But those times were the times I was ready to give up. This has been a hard road, and your support has really helped me get through this. You are much wiser than your years. Natalie, your smile has brightened my day so many times. I am so grateful to have your support, even when I had to take time away from you on nights and weekends to get this done. Girls, I hope you both realize that the world is waiting for you. You can do whatever you want as long as you are willing to work for it.

To my fellow math ed colleagues – Erin, Telashay, and Jenny, I could not have done this without your support and encouragement. It has been a wild ride. Thank you for letting me bounce ideas off you, reading my drafts, and supporting me when times got tough. I have enjoyed having you all by my side in this journey.

To my friend and officemate Hayley, thank you so much for all the support and encouragement you have given me. The hours we have spent talking to figure out the best way to help our students has been invaluable. I cannot express how much it has meant to me.

To my committee members, DeAnn Huinker and Kevin McLeod, thank you for pushing me. It makes my work so much better knowing that you will be reading it.

To Bo Zhang, thank you for igniting my love of educational statistics and measurement. I have learned so much from you in the classes you taught, and the feedback I have received writing this. Thank you for being a wonderful teacher and being on my committee.

To my advisor, Mike Steele, I can't thank you enough for all the guidance you have given me. Your support through this whole process has been so helpful. You were one of the toughest teachers I have ever had, and therefore learned more from you than I have from others. Thank you again for all you have done.

Last, I want to thank all my students. You are the reason I took on this project. My hope is that you are able to do what you set out to do for your life and learn a little along the way. Never give up trying – you will succeed – it might just take a little work to get there.

Chapter 1 Introduction

1.1 Personal Statement

I have been an instructor for developmental mathematics students for the last 14 years. Through all my years of teaching, I have noticed that many students struggle with some of the basic concepts of arithmetic and algebra because they do not have a conceptual understanding of these topics. Many of my students have been taught the procedures for integers, but not why those procedures work. They have been taught the "tricks" of mathematics, but not where those "tricks" come from. Then, when faced with multiple problems that look similar, they become confused. For example,

Solve: $\frac{3}{x} = \frac{4}{3}$

Add: $\frac{3}{x} + \frac{4}{3}$

Multiply: $\frac{3}{x} * \frac{4}{3}$

All three of these examples look very similar to students who can perform procedures but do not have a conceptual understanding of the operations that are being used. Many students will see all three of these problems and say that they need to "cross-multiply" because that is the procedure they were taught the first time they learned the concept. Often, the students never learn any concepts deeper than the procedure that will get them through the next test. Afterward, they promptly forget in what situations the procedure is used if they understood it in the first place, and they may even forget how the procedure works.

When I am teaching, I stress that we should not use the "tricks" and formulas until the students themselves understand where they come from. Sal Khan from Khan Academy uses the analogy of building a house in his TED Talk, *Let's teach for mastery – not test scores* (Khan, 2015). This talk discusses why students need to understand the basics that build the foundation of mathematics in order for their success in the future. His example of building the house starts with completing 80% of the basement, then 75% of the first floor, and 70% of the second floor. When the builders are trying to complete the attic, the entire house collapses because of all the gaps in the structure that was built underneath. As an instructor in developmental mathematics, I need to help students rebuild their mathematical houses from the basement up. This includes working with the existing knowledge they have and addressing the gaps and misconceptions to enable the students to add a solid foundation to what they have so they can move forward.

In addition to the struggles with conceptual understanding, many students at the developmental level have had little success in mathematics classes. Therefore, they do not want to even try in math class for fear that they will not do well. I feel my role as their instructor is to be more of a facilitator of their learning rather than a giver of knowledge. I try to give students the confidence to make mistakes and learn from these mistakes instead of giving up when they make a mistake.

Throughout my tenure as an instructor, I have had many students have light-bulb moments when they finally have an understanding of the mathematics they are learning. Once the students have this understanding, they gain confidence and can attempt harder problems than they thought they ever would be able to. My hope is to have these light-bulb moments for all my students. As they succeed in developmental mathematics, they can enter into credit-bearing mathematics with the understanding and skills they need to succeed. This will help them they will progress to their degree program and be better equipped to be on track to finish their degrees. In addition, as they gain a conceptual understanding of mathematics, they may decide to take more mathematics classes, STEM classes, or business classes. Taking more of these classes will open up many more career opportunities for them. Additionally, some of the students in the class are intended to be elementary teachers. As the future elementary teachers gain a conceptual understanding of the mathematics taught in this course, they help break the cycle of procedural instruction that led them to take the class in the first place.

1.2 Background

One of the primary ideals in the United States is that opportunities are open to everyone, no matter their background. “Providing access to postsecondary education to all people – even students who are not fully ready for college-level work – is a primary tenet of the U. S. educational system” (Brothen & Wambach, 2004, p. 38). Yet, only 20 percent of students who were referred to math remediation completed their college-level math course within three years (Bailey & Jeong, 2010). The completion rate for students who were referred to multiple developmental mathematical courses is even lower (Fong, Melguizo, & Prather, 2015). Students who are not fully ready for college-level work are still required to pay tuition and typically do not enter the full-time workforce while they are in college, so they are incurring debt and not earning money to pay back that debt. If the 80 percent or more of the students referred to math remediation do not complete their college-level math courses, they are not able to earn the degree they sought when they started postsecondary education, yet they will have incurred some of the cost of trying. This leaves disadvantaged students even more disadvantaged.

There is a nationwide discussion of developmental education reform. Currently, students entering colleges and universities in need of remediation do not complete their college-level coursework at the same rate as students entering college and not in need of remediation. According to Bailey & Jeong (2010), "[d]evelopmental education is designed to provide students with weak academic skills the opportunity to strengthen those skills enough to prepare them for college-level coursework" (p. 1). If this is the design of developmental education, students should be succeeding in their college-level course work as well as students who entered college without the need for remediation. Instead, according to Bonham and Boylan (2011), "courses which were originally designed to promote student academic achievement now often serve as barriers to that achievement" (p. 2).

Currently, many different aspects of developmental education are being looked at. They include methods of instruction (Cafarella, 2016; Fong et al., 2015; Kosiewicz, Ngo, & Fong, 2016; Rutschow & Schneider, 2011), classroom practices (Bonham & Boylan, 2011; Golafshani, 2013; Kosiewicz et al., 2016), and progression through the developmental sequence (Bailey & Jeong, 2010; Bonham & Boylan, 2011). Much of the work has found that addressing many of the problems and finding solutions to these problems increases student success. For instance, one problem in developmental education is student attrition (Bailey & Jeong, 2010), so to combat student attrition, providing students with accelerated or compressed sequences leaves students fewer opportunities to exit the sequence and therefore student retention increases (Cafarella, 2016).

There are also many studies that address the question of how to fill gaps in student knowledge for various topics. For instance, when addressing addition and subtraction of integers, using manipulatives has been found successful to facilitate understanding (Bolyard & Moyer-Packenham, 2012; Linchevski & Williams, 1999). It has also been shown that the use of number lines increases understanding (Bolyard & Moyer-Packenham, 2012). Both methods force students to model the operation in order to gain conceptual understanding. According to Sfard (1991), when students see the mathematical objects, they become real. This increases the ability for students to understand the three different meanings of negative numbers: unary, binary, and symmetric (Bofferding, 2009; Vlassis, 2008).

This study attempts to incorporate research-based instructional techniques to facilitate developmental mathematics students' success in their developmental mathematics course and beyond. The study incorporated the research about students placed into multiple developmental mathematics courses to determine where to focus the research-based instructional techniques. This study attempts to find effective methods of course design to facilitate developmental mathematics students' conceptual understanding of basic mathematics and beginning algebra, and students' belief in their own ability to do the mathematics.

1.3 Statement of Purpose

The purpose of this study is to determine the impact of incorporating best practices for teaching students basic mathematics and beginning algebra in the K-12 setting, along with growth mindset intervention, culturally responsive teaching, and what we know about classroom design for college students, for students who are referred to multiple developmental mathematics courses.

1.4 Problem Statement

Students placed in developmental mathematics at college have taken similar math classes at least once before in high school. While taking the class in high school (or before) they developed gaps and misconceptions in their understanding that prevented them from succeeding. Typical developmental mathematics classrooms do not address these gaps. Rather, they teach the material in the same way the students were taught in high school, only faster and in a more self-directed way.

1.5 Research Questions

1. How do the design changes affect student learning? In particular:
    a. How does targeted conceptual understanding instruction affect a student's conceptual understanding of and procedural fluency in integers and fractions and their operations?
    b. How do the design changes of the course compare success rates of current students to historic data from students before the reform took place?
2. How do the design changes affect students' attitude toward mathematics, and is there a change in discourse habits over the course of the semester?
3. What are the students' perceptions of the course design? In particular:
    a. What features of the course design do students believe is most important for their learning?
    b. What are the students' perceptions of learner-centered assessment system?
    c. What are students' perceptions of the learning opportunities designed to promote a growth mindset?

d. What are students' perceptions of learning opportunities to promote a conceptual understanding of integers and fractions and their operations?

1.6 Significance

This study will determine the ways in which changes to the design by incorporating best practices of K-12 topic teaching, growth mindset intervention, culturally responsive teaching practices, and a reorganization of the material of a developmental mathematics course influence opportunities for students to develop conceptual understanding of the topics taught in the course, thereby helping them enter credit-bearing mathematics after the semester and increasing their chances of obtaining a degree.

1.7 Research Design

This study was conducted using a design-based research approach. According to Moss and Haertel (2016), "the aims of the design experiment include both successful implementation and advances in theoretical understand[ing]" (p. 167). Thus, I tested whether specific approaches increased students' conceptual understanding, and also tried to discover what are the best approaches to teaching adult students the mathematics that they have been exposed to at least once. "Design experiments ideally result in a greater understanding of a *learning ecology* – a complex, interacting system involving multiple elements of different types and levels – by designing its elements and anticipating how these elements function together to support learning" (P. Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9).

The use of a design experiment approach affords me the ability to implement many different teaching practices and methods to determine if they are effective together. According to Cobb, et al., (2003), "[t]he theoretical intent, therefore, is to identify and account for successive patterns in student thinking by relating these patterns to the means by which their development was supported and organized" (p. 11). In addition, the authors go on to say that the researcher "must also specify the significant disciplinary ideas and forms of reasoning that constitute the prospective goals or endpoints for student learning" (p. 11). The experiment was be conducted over all the sections of the course that ran in the Fall of 2019. This enabled me to look at the different ways in which instructors enacted the design principles in the course and how this affected students' opportunities to learn.

To this end, I used a concurrent mixed-methods approach. I conducted a statistical analysis of pre- and post-tests to determine if there was a statically significant difference in students' understanding after instruction compared to before instruction. I compared results from the Attitude Toward Mathematics Index (ATMI) (Tapia, 1996) to determine whether students' attitude toward mathematics changed after learning in an active-learning environment with growth mindset messaging. I used a student survey to determine if themes emerged from student's perceptions. Finally, students were interviewed using two focus groups to determine their opinions of the class as a whole and the different methods of instruction used throughout the course.

1.8 Definition of Terms

The following terms will be used throughout the paper. These are the definitions that will be used moving forward.

Developmental mathematics courses are mathematics courses that are prerequisite mathematics courses to a student's gateway mathematics course. Developmental mathematics courses are designed to provide students with the skills and knowledge to succeed in their gateway mathematics course and do not carry any credits toward graduation. If a student is referred to *multiple developmental courses or levels*, they must pass more than one developmental mathematics course before they are able to enroll in their gateway course. A *gateway mathematics course* is a student's first credit-bearing mathematics course to begin their degree requirements.

This course is designed using a *holistic course design*. Developmental mathematics students are usually first-generation college students, struggle learning mathematics due to ineffective study skills, and have a fragmented mathematical knowledge. This course attempts to encompass all aspects of developmental mathematics students and move them toward mathematics success.

The course in this study has four underlying *design principles* (1) equitable environment, (2) provide learning activities that promote a conceptual understanding, (3) provide growth mindset and study skills instruction, and (4) develop a grading systems that emphasizes student learning (see Chapter 4 for a detailed discussion of each principle). The design principles are used to determine the appropriate *design features* and class activities that are implemented in the course.

A person's self-perception of themselves and their ability to learn is their *mindset*. A *growth mindset* is defined as a person's belief that they can learn with hard work and dedication (Dweck, 2008). A *fixed mindset* is defined as a person's belief that their abilities are inherent, and no amount of hard work will change their ability.

Learner-centered instructional design "is teaching focused on learning – what the student are doing is central concern of the teacher" (Weimer, 2013, p. 15). This instructional design places an emphasis on holding students accountable for their own learning, encourages students to work hard and learn from their mistakes, and encourages students to learn how to learn.

Active learning is an instructional practice in which the "view of the learner has changed from that of a passive recipient of knowledge to that of an active constructor of knowledge" (Anthony, 1996, p. 349). Students are encouraged to be actively engaged in the class activities to develop an understanding of the topics talked about in the class by doing problems and working in groups.

1.9 Limitations

This study was done across multiple sections of the same course. All sections have different adjunct instructors and teaching assistants, except one instructor taught two sections. All instructors were given the same training at the beginning of the semester and used the same lesson plans throughout the semester. While all the preparation was the same, I was not able to attend every class throughout the semester to ensure that the lessons were taught with fidelity. I made one classroom observation for each instructor, and those findings will be discussed in the results section. Additionally, some instructors are not comfortable with all the teaching methods described and made modifications to increase their comfort.

1.10 Delimitations

I have chosen not to study how the instructors interpret teaching using these different methods. I allowed them to modify the lesson plans to suit their comfort levels up to a point, but they had to inform me what they were changing. The instructor must have incorporated the redesign of the ordering of the topics, growth mindset intervention, and culturally responsive teaching practices to the best of their ability. If they chose not to incorporate the use of manipulatives when teaching fraction or integer operations, that is acceptable. The instructors were given instruction on how to use the manipulatives, but if they did not implement them with fidelity, it may have caused more harm than good. This is one place where students experience frustration, and if the instructor is unsure of how to use the manipulatives, it may have caused more confusion for students rather than increased understanding.

1.11 Summary

This study was a design experiment to determine if multiple interventions in a classroom environment and teaching methods affect developmental mathematical students' conceptual understanding of mathematics. The current state of developmental education at the university and college level and, in particular, the current state of developmental mathematics education will be discussed in Chapter 2. Additionally, Chapter 2 will discuss various teaching methods that enable students to develop a conceptual understanding of fractions, integers and their operations. Chapter 3 will discuss data collection and analysis. Chapter 4 will discuss the design principles of the course, including the research behind them, how they are implemented into the class and how they will be assessed. Chapter 5 includes both the quantitative and qualitative results. The final chapter, Chapter 6, will discuss the findings and look forward to further research areas.

Chapter 2 Literature Review

2.1 Introduction

"Providing access to postsecondary education to all people – even students who are not fully ready for college-level work – is a primary tenet of the U. S. educational system" (Brothen & Wambach, 2004, p. 38). Many students who enroll at an access university or a community college believe that graduation is one way to achieve a better career. This is indeed true, an adult with a bachelor's degree will earn about one-third more than an adult who starts but does not complete college, twice as much as one who has only a high school diploma, and will also have better health (Brock, 2010). College and university education allows students to explore new ideas, interact with people who are different from themselves, develop deep friendships, and make the transition into adulthood (Brock, 2010). In fact, "[a]ccess to higher education, it turns out, has increased substantially, although some racial and ethnic groups remain underrepresented. But success in college – as measured by persistence and degree attainment – has not improved at all" (Brock, 2010, p. 110).

One reason that students experience a lack of success is that they are admitted into a college or university and, through placement processes, are assessed as not being adequately prepared for credit bearing course work. Thus, many students must enroll in developmental education courses. For purposes of this study, a developmental education course is a course that is a prerequisite to a student's gateway mathematics course. According to Bailey, Jaggers, & Scott-Clayton (2013), "[t]he current systems of developmental education needs improvement" (p. 18). It is estimated that in 2010 the cost of developmental education was as high as $3 billion (Cafarella, 2016). This cost includes the time it takes for students to complete their developmental course work, the money spent on extra credits, and the opportunity costs associated with these factors (Bailey et al., 2013; Ngo & Kosiewicz, 2017). In addition, of the students who enroll in a four-year institution in the United States, one-fourth must enroll in developmental mathematics to start their education. Of the students who enroll in a two-year institution, 59% must enroll in developmental mathematics (King, Mcintosh, & Bell-Ellwanger, 2017). More importantly, many of the students that start being referred to multiple developmental mathematics classes do not pass their gateway course (Fong, Melguizo, Prather, & Bos, 2014). In the study done by Fong et. al, (2014) only 12% of the students that started in pre-algebra passed their intermediate algebra course and 7% of the students that started in arithmetic passed. This suggests that many of the students starting in developmental mathematics will not have the opportunity to have a chance at graduation and the career that is afforded them with a college degree.

In the pages that follow, I will discuss what we know about developmental mathematics students. Then I will discuss the current state of developmental education, with an emphasis on developmental mathematics education. Next, I will explore the current models of developmental mathematics education and their success rates for all levels of students. Finally, I will discuss the best practices for addressing these gaps in the current literature.

2.2 What We Know About Developmental Mathematics Students

Students in developmental mathematics share some commonalities. I will discuss the demographics of the typical student based on literature, their affective characteristics, and the types of mathematical knowledge they come into college with.

2.2.1 Student demographics

Developmental mathematics students are typically first-year students trying to make the transition from high school to college. As first-year students, they have to adjust to changes in their entire social network. This is true whether the student attends a local college or goes a long way to attend college. "Some students find ways to make this transition constructively and adapt to college, whereas others feel overwhelmed and unable to effectively meet the demands of their new roles" (Gerdes & Mallinkrodt, 1994, p. 281). In addition to their social network changing, this is the time when many students move away from home. They start purchasing their own groceries and household supplies (Gerdes & Mallinkrodt, 1994; Hicks & Heastie, 2008). All of this contributes to a high stress load for many first-year students.

At urban universities, first-year students have more challenges than the average incoming first-year student. These students are usually first-generation college students, are more likely to be placed in developmental education, are students of color (Crisp & Delgado, 2014), and have a lower SES compared to the average first-year student (Bailey, 2009). First-generation college students have significant obstacles to overcome. They are less likely than their peers to complete college-prep classes in high school even if they are prepared academically to succeed in these classes (Choy, Horn, Nuñez, & Chen, 2000). In addition, first-generation college students are less likely to know how to navigate college when they first start. They may miss important deadlines for admission or financial aid or may not be aware of services that are offered on campus. Finally, first-generation students may not know what services are available or even what questions to ask to find out what services are available.

Thus, students in developmental mathematics courses are usually at-risk students who may or may not know how to navigate the institutional practices at the college.

2.2.2 Student affective characteristics

In addition to student mathematical knowledge (discussed below), developmental mathematics students have other factors that contribute to their difficulties in mathematics success. In a study done by Acee, et al.(2017), the authors gave a survey to developmental mathematics students to determine perceived interferences to their success in college and math. The study found that only 5% of the students felt their math ability was inhibiting their math success. The other factors that students perceive as interferences to their success are course relevance and workload, and affective characteristics such as self-regulation, study method, motivation, time management, and stress/anxiety. In addition, Hall and Ponton (2005) found "a significant difference between the level of mathematics self-efficacy between freshman students enrolled in Calculus 1 and Intermediate Algebra" (p. 28). Students with low levels of self-efficacy tend to believe that they are unable to perform the tasks necessary for them to be successful. Thus, students with low self-efficacy and outside influences may not perform to the best of their abilities. Conversely, once developmental mathematics students "develop an interest in learning they are likely to surpass all expectations" (Higbee & Thomas, 1999).

2.2.3 Student mathematical knowledge

Students who are placed in developmental mathematics education have varying degrees of gaps in their knowledge and misconceptions in the knowledge they have. In part because it is difficult to measure and may vary widely from student to student, few studies have been done of what developmental mathematics students know (for exceptions, see Richland, Stigler, & Holyoak, 2012; Stigler, Givvin, & Thompson, 2009). In order for students to progress through the developmental sequence and be successful, they must fill these gaps and address the misconceptions. Stigler, Givvin, and Thompson (2009) did a study using the results of placement tests developed by the Mathematics Diagnostic Testing Project. They found that developmental mathematics students have difficulty simplifying fractions, operations with fractions, place values of decimal numbers, operations with exponents and square roots. They suggest that students have fragmented knowledge of the procedures in mathematics and misapply the procedures in a consistent fashion when faced with these types of problems. The authors go on to say "their knowledge of mathematical concepts may be fragile, their knowledge of procedures is firmly rooted – albeit in faulty notions of when and how procedures should be applied" (p. 14).

Colleges and universities use a variety of measures to determine if a student should be placed into developmental mathematics. These include a variety of placement exams, a student's ACT math sub-score, or multiple measures including a student's high school GPA and courses taken (Scott-Clayton, Crosta, & Belfield, 2014). At my university, a student is referred to basic math if they score a zero-composite score on the Wisconsin Placement Exam or score a 16 or below on the Math ACT. While an ACT math score is not a perfect measure of student knowledge, it is a common measure for many universities. Therefore, to have a common basis of students who are referred to basic math, I will use the ACT math score as a reference. According to the ACT College and Career Readiness Standards, students with an ACT math score of 13-15 (typically the students who are referred to basic math) know how to:

- Perform one-operation computation with whole numbers and decimals
- Recognize equivalent fractions and fractions in lowest terms
- Locate positive rational numbers on a number line
- Solve one- and two-step problems using decimals and money
- Extend patterns
- Solve one-step algebraic equations
- Calculate the average of a list of positive whole numbers
- Extract one relevant number from a basic table or chart, and use it in a single computation. ("ACT College and Career Readiness Standards," 2017)

Looking at this list, it appears that students needing multiple developmental mathematics classes may have gaps or misconceptions in:

- Number sense and properties involving even/odd numbers and factors/multiples
- Categories of numbers and what they mean and imply (natural, integers, rational, irrational, real and complex)
- Solving arithmetic problems involving rational numbers and percentages
- Relating graphs to situations
- Integer operations
- Fraction operations
- Exponent properties
- Polynomial operations
- Solving linear equations and inequalities

- Substituting unknown numbers for unknown quantities

Looking at the two descriptions, it indicates that students who are referred to multiple developmental mathematics classes may have a fragile knowledge of basic mathematics, and their knowledge may be based on procedures that are likely to be misapplied in practice.

2.2.3 Conclusion

A number of factors affect the performance of students entering a developmental mathematics classroom. All these factors must be taken into account when designing a course for developmental mathematics students to be successful. As students find success in their early math courses, they may have a renewed confidence in their mathematics abilities (Mireles, Acee, & Gerber, 2014). When this happens, they will have more opportunities for their careers after college compared to career options of students who have minimal mathematics classes.

Next, I will discuss the current structures of developmental education with an emphasis on developmental mathematics education. Then I will discuss the instructional designs from K-12 and college that support the fragmented understanding of students who are referred to multiple developmental mathematics classes.

2.3 Developmental Education

Students throughout the country are encouraged to enter colleges or universities so that they have more career opportunities. Many of these students are under-prepared for college-level course work. According to Stan Jones, president of Complete College America, as quoted in *The Chronicle of Higher Education*, "for many of these students, a remedial course is their first college experience, as well as their last" (Mangan, 2014). Jones goes on to say that community colleges have been able to diversify their first-year classes, “but if you fast-forward to graduation day and look who’s on the stage, they’ve lost a lot of that representation.”

Developmental education is designed to enable students who enter college under-prepared for college-level courses to gain the necessary skills to succeed (Bailey & Jeong, 2010; Goudas & Boylan, 2012). Since many students enter college in need of developmental education, and at varying degrees of support, there are many different developmental education courses. Any course that is considered a prerequisite for an entry-level credit-bearing course is considered a developmental course for the purpose of this study; in some places, developmental courses and remedial courses are synonymous when discussing courses at colleges and universities. These courses usually offer no credit toward graduation. For the purpose of this study, the number of these developmental courses a student has to take in the same subject is considered the number of levels below credit-bearing the student is placed. Therefore, if a student is referred to basic math and beginning algebra, they are referred to two levels of developmental mathematics. For the purpose of this study, students that are referred to two or more levels of developmental mathematics will be considered to need multiple levels of developmental mathematics. According to Bailey & Jeong (2010), “two-year colleges nationwide reported to offer, on average, 3.6 remedial courses in math and 2.7 remedial courses in reading” (p. 2). Navigating through all these courses for a student who may not be academically prepared can prove challenging (Bailey & Jeong, 2010). When students start at the lowest levels of developmental education, many choose not to attend college or take credit-bearing classes against advice of faculty and advisors (Valentine, Konstantopoulos, & Goldrick-Rab, 2017). In many cases, taking credit-bearing classes against advice leads to not passing the credit-bearing class. In addition, there are many factors other than simply passing a sequence that contribute to the likelihood of completing the sequence, such as financial obstacles, family obligations, and even health complications (Fong et al., 2014). Thus, for a student who starts at the lowest level of remediation, it is extremely difficult to complete the progression through the sequence of their developmental program and the corresponding college-level class. "Just as there are myriad needs that students bring to developmental education programs, there may be more than a few ways to meet those needs" (Brothen & Wambach, 2004).

Progression through developmental sequences is complicated, time-consuming, and expensive. Even though all courses in a developmental sequence are meant to prepare students for the next course in the sequence and finally their first college-level course, many students fail to complete the sequence because they fail to enroll to begin with or stop mid-sequence (Bailey & Jeong, 2010; Bonham & Boylan, 2011). "When considered from the beginning of the sequence, only 20 percent of students referred to math remediation and 37 percent of those referred to reading remediation completed a gateway course in the relevant subject area within three years" (Bailey & Jeong, 2010, p. 3). A gateway course is considered the course that satisfies the minimum graduation requirements at a particular college or university. Students who are referred to more levels of remediation are the least likely to complete their sequence through developmental education and into college-level courses (Bailey & Jeong, 2010).

In addition, many students who have been referred into a developmental sequence have taken and passed all their high school coursework. They enter their college experience surprised to find out that they are not college-ready according to their placement exam or some other measure of college readiness that their college or university uses. In 2013, Florida legislators decided that any student who graduated from a Florida high school was (1) not mandated to take a placement exam and (2) was not mandated to take a developmental education course (Smith, 2015). The purpose of this law was to save students money and encourage them to stay in college. *Inside Higher Ed* quotes Lenore Rodicio, provost for Miami-Dade College's academic and student affairs, as saying "[t]he ramifications are multiple. In the simplest case, the students retake the course, but retaking the course if you still don't have the proper preparation just means more money wasted" (Smith, 2015). Thus, in the end, students who choose to take a college-level course may not have the support they need to succeed instead of taking a developmental course that prepares them for that college-level course. Therefore, they may end up spending more money and time instead of less.

Currently, there are several studies looking at the effects of developmental education for students who place very close to the cut-off for college-level mathematics courses. These studies show that for the students who are close to the cut-off and are placed into developmental coursework, there are negative outcomes (Valentine et al., 2017). This includes more money spent, more time in remediation, and a lower likelihood of persistence and degree attainment. (Melguizo, Bos, & Prather, 2011; Ngo & Kosiewicz, 2017). Ngo and Kosiewicz state, "[t]he puzzle is that the very intervention that is aimed at preparing students to be successful in gateway college-level courses may at the same time be an obstacle and deterrent to their persistence in college" (2017, p. 268). These studies address the students who barely missed placing in college-level courses. Many times, students are not close to the cut-off score and may need to take two or more courses in a sequence before they fulfill the prerequisite for their gateway courses. When these students are required to take two or more courses, there are many factors that contribute to their lack of success in their gateway course, including motivation and abilities (Brothen & Wambach, 2004), failure to enroll in the next course (Fong et al., 2014), perception of taking developmental courses (Smith, 2015), and family and life constraints. There is a gap in the research about how best to serve these students.

Universities and colleges are currently admitting many students who are under-prepared for college-level coursework. These students hope to attain their degrees and thus have more opportunities for careers and advancements in their lives. In addition, people who receive a bachelor's degree typically have better health and fulfillment in life. Unfortunately, even though these students are admitted to universities and colleges, many of them do not complete their degrees. Therefore, they end up without the opportunity for a better career or job advancement. In addition, they end up with the costs of the education they could not complete and struggle to pay for that education. Therefore, research needs to be done on how best to serve all students who are admitted to universities and colleges, so they have the support they need to succeed.

2.4 Developmental Mathematics Education

In most colleges, math is the gateway for degree fulfillment, so students who struggle to complete their math requirement are not able to progress in the degree plan and graduate. According to Valentine, Konstantopoulos, & Goldrick-Rab (2017), "[m]ath is the most common subject in which remediation is needed, with participation rates (about 15%)" (p. 807). In addition, the traditional developmental sequence of mathematics hinders students from entering and completing their college-level mathematics courses (Boylan & Bonham, 2007; Cafarella, 2016). The good news is that research shows that the students who do complete their developmental mathematics sequence are just as successful in their college-level mathematics course as students who were not referred to developmental mathematics (Bailey, 2009; Bailey & Jeong, 2010; Bonham & Boylan, 2011). Therefore, a deeper look needs to be taken to determine the best way for students to complete their developmental mathematics sequence.

Developmental mathematics traditionally includes entry points that start at basic arithmetic, pre-algebra, beginning algebra, and intermediate algebra (depending on the college). These courses can be split up in numerous ways to be a course sequence that includes between two and four courses. Thus, students who are referred into basic arithmetic can have as many as four semesters in a developmental course sequence before they can start their college-level course work. Since developmental courses usually are not counted toward degree credit, this traditional sequence of developmental coursework takes time and resources and may discourage students from completing their degree (Bailey et al., 2013).

Students who are placed in basic arithmetic may have a fragmented understanding of mathematics from as early as fourth through seventh grade. Students who are placed in beginning and intermediate algebra may have a fragmented understanding of middle school and high school mathematics. The developmental mathematics classes that students take are usually still being taught in the same topic progression as the students had in elementary, middle, and high school (Kosiewicz et al., 2016). This is not successful for many students, as approximately 7% of the students who start in basic arithmetic pass intermediate algebra (Fong et al., 2014). There are some models of developmental mathematics education that have shown some effectiveness of students progressing through the sequence and successfully completing their college-level coursework. Therefore, a look at the different models is necessary to determine whether some models are more effective at helping students progress through their gateway math courses than others.

2.4.1 Current Course Structures of Developmental Mathematics

There are many different course structures of developmental mathematics education in place around the nation. Some of the structures are more successful than others. The most promising structures have students spend the least amount of time in the developmental sequence (Brothen & Wambach, 2004; Cafarella, 2016). Thus, finding ways to shorten the sequence of developmental education by using a corequisite model or accelerating students has shown the most promise. Ngo & Kosiewicz found that "[d]espite the potential benefits to learning of increasing time in math, extending math over two semesters inherently increases the need for students to make persistence decisions at exit points and increases the costs associated with completing math remediation" (p. 295). The current models are described below.

2.4.1.1 Increasing the time in developmental mathematics sequence. One of the first models of developmental mathematics sequence was to increase the time spent in the sequence so that students would gain a better understanding of the material they struggle with. Thus, some colleges assigned students to an elementary algebra course that extended for two semesters instead of one semester. The extended elementary algebra course started them with the same content day one as the one-semester elementary algebra course but it proceeded at a slower pace (Ngo & Kosiewicz, 2017) Ngo & Kosiewicz found that "the practice of extending math courses in this way reduces the likelihood that students will complete the developmental math sequence and persist in college toward credential attainment" (p. 294). In addition, studies done in middle school and high school find that adding a second algebra class is least effective for students with the weakest math abilities (Nomi & Allensworth, 2009) and any gains that were seen were not sustained in subsequent years (Taylor, 2014). Therefore, increasing the time gives students more time to drop out of the sequence and may not increase their success as they move through the sequence.

When increasing time in the developmental mathematics sequence, there is more opportunity to exit the sequence. At each exit point, students must decide if it is a benefit to them to continue their progression or stop. Many students in the developmental sequence are non-traditional, meaning they are older than 25, parents (possibly single parents), and working, in addition to going to school. Thus, at every exit point, as many as 50% of the students choose to exit the sequence and many do not come back (Fong et al., 2015).

2.4.1.2 Corequisite remediation. Corequisite remediation is designed to provide students who are one level below college-level mathematics an opportunity to enroll directly into college-level mathematics with support provided as needed. Some institutions are pushing for all developmental mathematics students to enroll into co-requisite courses, even though the course is only designed for students who are referred to one level of developmental mathematics. Students typically enroll in the college-level mathematics course and a support course that is between one and three credits. Sometimes the developmental students are integrated into the college-level mathematics course with students who are not in need of the support course (Bracco, Austin, Bugler, & Finkelstein, 2015). This model of remediation integrates developmental education into the college-level courses (Brothen & Wambach, 2004). Otherwise, the developmental students have their own college-level course that includes the support course (Bracco et al., 2015). This helps "developmental students with one specific course's assignments, much like tutoring does" (Goudas & Boylan, 2012, p. 8).

According to Stan Jones of Complete College America, "[b]y providing remediation as a corequisite – not a prerequisite sequence that sets students back – attrition is reduced, and long-term academic success becomes more likely" (Jones, 2015). He goes on to state that the goal is to have students spend less time on coursework that does not count toward their degree. Corequisite remediation also provides fewer exit points (Jones, 2015) and thus more opportunities for success (Fong et al., 2015).

Corequisite course models provide the added benefit of starting students in their college-level coursework from day one. Students are provided with just-in-time remediation of the relevant material and therefore, the redundancy between subsequent courses is reduced or eliminated (Hern, 2012). This gives students the purpose of learning the prerequisite material, instead of learning the material out of context in a previous course.

The corequisite model has been shown to be most effective for students who are referred to only one developmental class before they fulfill their prerequisite for college-level mathematics. Because corequisite remediation has shown at least minimal success at all levels of learners, Complete College America recommends that all developmental courses should be cut from colleges and replaced with corequisite model education (Jones, 2015). But, according to Goudas & Boylan (2012), "this fundamental movement away from prerequisite remediation is based mostly on research conclusions which are taken out of context and misapplied" (p. 8). For instance, Tennessee has replaced all their developmental mathematics courses with corequisite courses. This shift has proven extremely successful for students who have an ACT math score of 19 or above. The students who have an ACT math score of 17 and 18 show some success, and the students who have an ACT math score of 16 or less have very little success ("TBR CoRequisite Study - Update Spring 2016," 2016). These students with an ACT math score of 16 or less typically need two or more prerequisite courses before they are able to take a college-level course. Thus, if they are referred to a course that provides in-time remediation support for the college-level course material, they may not be receiving instruction on the material where they are in need. Thus, the students may not be receiving the instructional support they need to succeed. Georgia has done a similar study on corequisite courses (Denley, 2017). Georgia found that students enrolled on a corequisite course had a pass rate of 63% overall, and students with an ACT Math score of 16 or less had at least a 56% pass rate. Georgia has three different approaches to the corequisite approach and did not separate the pass rates for each approach. Therefore, it is unclear what the pass rate was for the college algebra approach to corequisite instruction. Thus, the corequisite model shows promise for all students but students with ACT Math 16 or less may benefit from other models of instruction also.

2.4.1.3 Modular curriculum. In a modularized curriculum, the mathematics curriculum is divided into different modules of topics. Typically, the modules are one credit each and students can take up to four modules in one semester. The modules can be instructor-led or self-paced (Bickerstaff & Fay, 2016; Rutschow & Schneider, 2011). Students are required to complete only those modules that are identified areas of weakness for them. Thus, students work through customized intervention modules targeted to address the specific competencies or skills that they need to improve – not on content areas that they have already mastered (Bickerstaff & Fay, 2016; Bracco et al., 2015; Rutschow & Schneider, 2011) Students can address the areas where they have gaps and may not need to take an entire two-semester or longer sequence of developmental mathematics. Conversely, this approach provides many different exit points for students to not complete their sequence. One of the drawbacks of this approach is that a student may complete the diagnostic test and determine that they need to complete modules 1, 4, 5, and 6. According to the test, the student has shown mastery of modules 2 and 3. The material in modules 2 and 3 requires students to have mastery of module 1. Thus, can it be assumed that the test was correct if the student does not know the prerequisite material for a module, but they have shown mastery of the material in that module. It may be that the assessment to determine what modules a student needs to take does not account for prerequisite knowledge in their different question types.

There are drawbacks for students working on only targeted modules. According to Bickerstaff & Fay (2016), it is very difficult for instructors to cover a large volume of content in a short amount of time. In fact, “when accounting for the class periods when tests are administered, some colleges have as few as 6 days of instruction per module in stand-alone courses” (p. 22). The authors go on to point out that unless students can register for the same instructor, it is difficult to build a student-teacher relationship. This student-teacher relationship is a fundamental principle of culturally responsive teaching (Hammond, 2015). In addition, instructors have difficulty drawing connections throughout all the material for students to understand the reasons for the material being taught. Instead, students see a module on integers, a module on fractions, and a module on decimal numbers and percentages without the inherent connections between these concepts. In the shell courses, where students can register for up to four modules at once and are taught in a computer lab, students with questions during class may wait for long periods of time to obtain assistance from their instructor. In addition, it is difficult for students to ask help from other students because everyone is working on different material at their own pace. In fact, the *MAA Instructional Practices Guide* (2018) suggests that math should be taught in a collaborative classroom for effective student learning. Students should work together discussing mathematics and learning from each other (National Council of Teachers of Mathematics, 2014), but this is difficult to accomplish with as little as six instructional days for a unit.

Since many students who start in the first modules of the curriculum struggle with learning the mathematics involved, “modularized structure enables students who require more time to master content to repeat or spend additional time working on a module, with less of a time penalty than in previous developmental course structure” (Bickerstaff & Fay, 2016). Students must repeat only a four-week course instead of a semester-long course. Conversely, this adds multiple exit points to the sequence and therefore, many students exit the system and do not complete their developmental coursework. In addition, this adds to students’ confusion when registering for courses, because there is no single path through their developmental coursework.

Bickerstaff & Fay (2016) found that in Virginia, the system-wide pass rates for stand-alone modules ranged from 59 to 76 percent. Not surprisingly, the lower pass rates were in the first modules with the most remedial material. In the shell courses, 42 percent of the students trying to complete four modules in one semester only completed one, two, or three modules. Therefore, they received a non-passing grade on their transcripts. This has an impact on their academic progress and may impact their financial aid eligibility. Finally, according to Bickerstaff

& Fay, for students "who placed into the module 1-5 range, only 18 percent attempted a college-level math course" within one year and "for students who placed into the module 6-9 range, that figure was 57 percent; and for students who placed out of all modules, it was 86 percent" (p. 31). Therefore, similar to the results from Tennessee, students referred to the most prerequisite developmental courses struggle to pass all their developmental coursework.

2.4.1.5 Emporium model. The emporium model started at Virginia Tech with the idea that the best time for learning mathematics is when the student wants to, rather than a time that the course catalog determines (Twigg, 2011). Students are enrolled in a math course that requires them to access their course materials in a computer lab. At Virginia Tech, the computer lab has 500 workstations and is open 24/7. Some other campuses that employ the emporium model are the University of Alabama's Mathematics Technology Learning Center, which has 240 computers and is open 72 hours per week, and the University of Idaho, which has 72 computers, in pods of four, and is open 80 hours per week. Multiple sections of a course are combined into one large course structure. Some of the labs are set up so that students can collaborate or have a private room for additional tutoring. The students work with online materials and computer-aided software for their instruction and immediate feedback. The idea behind the model is that "Students learn math by doing math, not by listening to someone talk about doing math" (Twigg, 2011, p. 26). In an emporium model, students do the math but often in a disconnected fashion taught by a computer. There may be personalized instruction from faculty, graduate TAs, and undergraduate assistants (Twigg, 2003, 2011), but this is usually on a problem-by-problem basis and connections are not made between different topics. For students to develop a deeper understanding of mathematics, they need to see the overarching connections between mathematical structures (AMATYC, 2018; National Council of Teachers of Mathematics, 2014).

Twigg (2011) suggests that, for the emporium model to be most effective, students need to be required to participate in the computer lab experience and be given points for doing so. “Students participate more, score higher, and spend longer on course activities when credit is at stake” (Twigg, 2011, p. 27). When students are given an open-ended approach to learning, they tend to wait until tomorrow to work and tomorrow never comes.

This model is cost-effective because it combines faculty, graduate teaching assistants, and undergraduate assistants with the lab to help the students with their online homework when they get stuck (Twigg, 2003, 2011). Some campuses even have students contact undergraduate assistants with course progress and technical issues, which frees up faculty to concentrate on identifying the students who need the most help and working with those students individually (Twigg, 2003). This can lead to multiple instructors for one student, therefore students have trouble building the student-teacher relationship that can create an equitable environment where students learn best (Hammond, 2015; Zull, 2002). In some cases, this is an effective strategy if students can see the similarities among different sets of instructions. From my experience in a modified emporium model, it can be disastrous for students who see each set of instructions as mutually exclusive. Students end up asking only one instructor for assistance and sometimes waiting a long time for that assistance. According to Twigg (2011), these models show promise for students passing through their developmental mathematics courses. The studies that are mentioned in the article are mostly classes in college algebra and above. The one study that does use the emporium model for developmental education is the Tennessee study mentioned in the corequisite section. Therefore, based on the results above, the students with Math ACT scores lower than 17 do not benefit from the emporium model. Zavarella and Ignash (2009) found that students "enrolled in developmental courses may not be cognizant of their particular learning needs or have misconceptions of computer-based instruction" (p. 9). Thus, these students may have a harder time completing their developmental coursework in an emporium style course.

2.4.1.6 Mentors and supplemental instruction. Another model of instruction for developmental mathematics is providing mentors and supplemental instruction. According to Burley, Butner & Wilson (2009), "[d]evelopmental education programs need to cocoon students in culture of success with significant peer and mentor support" (p. 37). Supplemental instruction and mentors can provide this support. In supplemental instruction, students who have succeeded in the course are given training in tutoring and employed as supplemental instruction (SI) leaders. The SI leaders attend the class and help if needed. After class, they have specific times during the week where they provide tutoring and instruction on the concepts that students were having trouble within class. The SI leaders also typically provide additional exam reviews (Wright, Wright, & Lamb, 2002). According to Wright, Wright & Lamb (2002), it is important for the instructor to foster SI participation. For instance, the instructor can give out a worksheet and the answer key to the SI leader, the instructor can work with the SI leader to prepare handouts that are only available from the SI leader, or the instructor can encourage the SI leader to make short presentations in class.

Supplemental instruction can be used in conjunction with corequisite remediation. This provides additional support for students in the developmental course. According to the MDRC report, for supplemental instruction to be most successful, students must self-select to use the supplemental instruction (Rutschow & Schneider, 2011). Then, "when using large data sets to compare students who received supplemental instruction and those who did not, the intervention was found to have resulted in higher grades, lower course withdrawal rates, higher GPAs, and higher rates of persistence and graduation" (p. 47-48). Another factor in the success of supplemental instruction is instructor support (Wright et al., 2002). Therefore, supplemental instruction that is self-selected by students and supported by instructors is effective in helping students progress through their developmental sequence, but students most in need of support may not seek out supplemental instruction.

2.4.1.7 Accelerating/compressing time in the developmental mathematics sequence. The goal of accelerating the developmental mathematics sequence is to reduce the time that students spend in remediation. Typically, acceleration involves students enrolling in a course that meets more often than a typical class but lasts for only one semester. This reduces the number of exit points that students have to exit the course sequence (Bracco et al., 2015). According to Cafarella (2016), although acceleration and compression may be best for some students, it is not a universal best practice. Hern (2012, p. 64) states, "that although low-scoring students are at higher risk than other students, they are not better served by a slower, multi-semester sequence." Levin points out that "a better strategy for success is not to slow down their development and learning through repetition of the lowest skills, but to incorporate those skills into more meaningful educational experiences that will accelerate their growth and development to bring them into the academic mainstream" (as cited in Bailey, Jeong, & Cho, 2010, p. 269). These findings suggest that, while changing the course structure does show promise for some students, incorporating these changes, along with changes in instructional practices may be more beneficial.

Accelerating a course sequence enables colleges to put two or more developmental mathematics courses together into one semester-long course. Typically, these courses have the same number of credit hours as the traditional sequence. The accelerated course will meet more times during the week or have a longer class session for the same number of days per week. These longer blocks of instructional time enable instructors to provide a number of different activities and group work to enhance the instruction (Jaggars, S. S., Hodara, M., Cho, S., & Xu, 2015; Sheldon & Durdella, 2010). The benefits of active and collaborative learning will be discussed later in this literature review.

2.4.1.8 Additional success course. Student success courses are intended to add additional support for students in developmental courses. These courses can range from discipline-specific courses to generalized student success. They are normally offered as stand-alone classes that offer a minimal amount of credits toward a degree for developmental students (Rutschow & Schneider, 2011). The courses offer students extra support, practical practice learning study skills and note-taking methods, individualized tutoring, and study groups (Brock, 2010). According to the MDRC report, "[s]tudent success courses have become one of the most popular support interventions among community colleges seeking to improve developmental-level students' achievement" (Rutschow & Schneider, 2011, p. 50). This same report states that student success courses have seen promising results showing positive gains in credits earned and progression through the developmental sequence.

2.4.1.9 Remediation before college entry. There are two types of remediation that occur prior to college entry. First, students can be identified in high school as intending to attend some form of higher education and also having a need for remediation. The high school works in conjunction with an area college or university to offer duel enrollment. The MDRC report finds that throughout the country these programs have affective benefits for students (Rutschow & Schneider, 2011). Many times, these programs will offer students the ability to enter into college-level coursework if they successfully complete their dual enrollment program successfully.

The second option for students to complete their remedial work before college entry is to participate in a summer bridge program. The summer bridge programs offer intense remediation before students start classes (Brock, 2010). Many summer bridge programs are three to six weeks in length. They can be in-person, online, or a hybrid of the two. They can be formalized classes or self-paced, computer-aided programs. Once the summer bridge program is complete, students can re-test to see if they have increased their placement level. According to the MDRC report, summer bridge programs "show promising improvements in students' study skills and college readiness in math and reading" (Rutschow & Schneider, 2011, p. ES 3).

2.4.1.10 Conclusion. There are many different models of developmental mathematics education. Many of them offer promising results for different kinds of students. There are commonalities among the successful models, though. The successful models have students meeting more days a week than in a traditional course. This includes in-time remediation in a corequisite course or more time in an accelerated course. Students in a summer bridge program also meet more often. This type of model requires students to do mathematics more often. The act of doing mathematics more often could be one of the factors contributing to the success of the model. Another aspect that may contribute to the success of these models is the reduction of exit points in the sequence. Since there are fewer exit points, there are fewer opportunities for students to exit the sequence and thus not complete their developmental coursework.

Unfortunately, students who are referred to multiple prerequisite courses of developmental mathematics education have the most difficulty completing their developmental mathematics sequence. These students are admitted to colleges and universities and therefore should be given an opportunity for success. But, developmental mathematics has been taught in the same sequence, using the same instructional practices for many years (MAA, 2018; Stigler et al., 2009). We know that students in developmental mathematics rely on a fragmented understanding of procedures and have difficulty applying those procedures in the correct context, and students referred to multiple prerequisite courses have multiples opportunities to exit the sequence and not complete their coursework. I will next look at learner-centered instructional design practices. According to Weimer (2013), learner-centered instructional design "is teaching focused on learning – what the students are doing is the central concern of the teacher." (p. 15)

2.5 Instructional Practices

In traditional mathematics instruction at a university or college, instructors lecture from the front of the room and students take notes of exactly what the instructor writes on the board. In fact, many studies have found that in the United States, emphasis is placed on practicing procedures and rote memorization (MAA, 2018; Richland et al., 2012; Stigler et al., 2009; Stigler, Givvin, & Thompson, 2010). This approach leaves many students at a disadvantage because it emphasizes students' rote memorization and does not involve them in the learning process. Some instructors ask for input as they are lecturing, but many tell students what to do and how to do it. As you walk through the halls on many campuses and look in the classrooms where instructors are lecturing, you will see students sleeping or on their phones and many not actively paying attention to the instructor. If students are not asked to make sense of the material, they will have difficulties performing the high-order levels in Bloom's taxonomy (Boston & Smith, 2009). Mathematics instruction needs to move away from the traditional lecture format and start using the class time for different activities. Moyer and Jones say, "Promoting autonomous thinking in students requires a shift in the mathematics teaching and learning routine and a willingness to think and make sense of mathematics for themselves" (2004, p. 29). By promoting autonomous thinking and having students participate in authentic activities, they will become better prepared to enter the workforce. Once students start their careers, they will be asked to apply the mathematics they have learned and find a solution. I will discuss some classroom strategies that help the students become their own constructors of knowledge.

2.5.1 Active learning

Active learning is an instructional practice in which the "view of the learner has changed from that of a passive recipient of knowledge to that of an active constructor of knowledge" (Anthony, 1996, p. 349). In an active-learning classroom, learning activities commonly include problem solving, small group work, investigational work, and collaborative learning. Many organizations are recommending that instruction at the college level include active-learning techniques (AMATYC, 2018; CBMS, 2016; MAA, 2018; National Council of Teachers of Mathematics, 2014).

According to Stipek, et al., "[t]he role of the teacher is to support and guide this constructive process rather than to transmit discrete knowledge" (Stipek, Givvin, Salmon, & MacGyvers, 2001, p. 214). Students then work in class to understand mathematics, instead of passively sitting and trying to make sense of their homework. According to Smith III (1996), "[t]eaching by demonstration and practice is no longer acceptable, because students cannot learn mathematics as passive listeners" ((Smith III, 1996, p. 388). When students work to actively construct their mathematical knowledge, they can retain more and understand the connections to the real world. "Students actually learn math by *doing math* rather than spending time listening to someone talk about doing math" (Bonham & Boylan, 2011, p. 4, emphasis added).

Active learning helps all learners by positioning each student as an active participant in learning. Since the students are engaged in the learning process, they tend to be more motivated to do the learning (MAA, 2018). The position statement on active learning from the Conference Board of the Mathematical Sciences (CBMS) (2016) points out that "active learning confers disproportionate benefits for STEM students from disadvantaged backgrounds and for female students in male-dominated fields." Thus, by using an active-learning classroom, the at-risk students are able to find the success they need to proceed through their mathematical sequence. [The CBMS is "an umbrella organization consisting of 18 professional societies all of which have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences." (CBMS, n.d.). These professional societies include

American Mathematical Association of Two-Year Colleges (AMATYC), American Mathematical Society (AMS), Association of Mathematics Teacher Educators (AMTE), Mathematical Association of America (MAA), National Council of Teachers of Mathematics (NCTM), and many more.]

There are two major challenges when implementing this model of instruction. The first is instructors' comfort and ability in giving some of the control in the classroom to the students. Most college and university faculty and instructors are experts in their field, but few have any formal instruction in pedagogical practices. Therefore, it can be very uncomfortable for instructors to step away from the board, and let students answer questions in their own way. According to Smith III (1996), "[w]hen reform shifts the emphasis in mathematics content and pedagogy from learning rules and procedures to understanding, explanation, and problem solving, teachers are often at a loss to know what and how to teach" (p. 395). When teaching in an active-learning classroom, teachers have to be willing to allow the students to construct the mathematics for themselves. This includes changing the classroom norms that many students have been accustomed to for quite a while (Wood, Cobb, & Yackel, 1991). Because of this, instructors may not know what the classroom would look like, and if they do know what it does look like they may not know how to get there (Hufferd-Ackles, Fuson, & Sherin, 2004). This stepping into the unknown is a daunting task for many teachers.

The second major challenge is the students' comfort and willingness to participate in an active-learning classroom. Many students, especially at the developmental level, are not comfortable with their abilities in mathematics. They are reluctant to make mistakes – especially in front of their peers. Instructors need to learn how to handle students' answers, whether correct or incorrect. This practice will encourage the student-centered learning environment (MAA, 2018) and open the door for students to learn from their mistakes and others (Boaler, 2015). The *MAA Instructional Practices Guide* (2018, p. 5) states "from an equity stance, one of the most powerful ways an instructor can build community and student confidence is to reframe errors." Smith III (1996), supports this when he says, "[s]tudents can be vocal about their discomfort with and lack of belief in, new classroom mathematics and openly resist teachers' efforts to experiment and change" (p. 396). When implementing a learner-centered approach to teaching, the instructor needs to let the students be uncomfortable, but not overly frustrated (Weimer, 2013). While it is difficult for students and teachers to change to an active-learning classroom, once the change has been made the benefits far outweigh the initial discomfort.

It is very true that "[p]romoting mathematics learning environments where students construct meaning requires major shifts in the sets, scripts, and roles of teachers and students" (Moyer & Jones, 2004, p. 29). The adjustments can be difficult for students and instructors. The benefit is that students learn mathematics by doing the mathematics (Bonham & Boylan, 2011). Instructors are able to judge what their students are understanding and what they are not understanding and adjust their teaching to the needs of the students (Wang & Cai, 2007). Wang and Cai point out that "to reach this fundamental goal, the teachers argue, effective lessons should have students interested, motivated and involved" (p. 324). While setting up the course for active learning may be difficult, seeing the benefits in the students make the transition worthwhile.

2.5.2 Collaborative learning

Another technique for instructors to facilitate student learning is through collaborative teaching. This can come from group work in the classroom, paired board work, and even group projects. "Instructors will need to create a classroom environment where students feel accountable both as individuals and as members of the classroom community of learners" (MAA, 2018, p. 1). By building this classroom community, teachers are able to foster learning among all students (Hammond, 2015). Instructors need to recognize the differences among their students and understand how these differences shape our students and their learning (Hammond, 2015; MAA, 2018).

Collaborative learning enables students to help each other learn. Students are able to develop a deeper conceptual understanding as they explain the concepts to each other. "These collaborations facilitate learning to form logical arguments and, as a result, students are able to tackle more difficult problems" (MAA, 2018, p. 21). These problems can include problems that make connections among multiple representations in mathematics. This will enable students to "grow in their appreciation of mathematics as a unified, coherent discipline" (National Council of Teachers of Mathematics, 2014, p. 29).

In addition to offering students richer problems that help develop their conceptual understanding of mathematics, collaborative learning builds a classroom community. This community connects students to their instructors and fellow teachers. The connections that students have given each of the other students a responsibility to themselves and each other. "Students care when they believe that other people care about them. They are less likely to drop out" (McKay, 2015, no page).

2.5.3 Flipping the classroom.

One way to free up class time for active-learning activities is to flip the classroom. In a flipped classroom, students access the learning content outside of the classroom and do much of the learning inside the classroom. Outside of class, students are required to read the textbook or watch video lectures. Students are able to take their time, rewind parts of the video or re-read the section they did not understand, and take detailed notes (Larsen, 2015). Then, when students are in class, the instructor can facilitate their learning with activities, discussions, group work, or formative assessments. Class time is facilitated by the instructor based on student needs (Larsen, 2015). In a flipped classroom, the instructor "recognizes that scaffolding and guidance is needed in order to maximize the likelihood of success and to maintain reasonable levels of motivation" (MAA, 2018, p. 42).

This type of class promotes autonomous learning for every student. Students are in control of their lecture and their note-taking pace. Students who miss classes because of illness are able to learn on their own outside of class and catch up to where the class activities are (Larsen, 2015). In the study that Larsen (2015) performed, she had some students who were able to complete the class by only watching the videos and working through the homework. Therefore, the students who needed extra help in the classroom could receive the help they needed. The students who were able to watch the videos and make sense of the homework on their own were able to complete the course without coming to class.

Using a flipped-classroom approach enables instructors to position the material where it is most accessible to the students. Before each class time, the instructors can give students the basic facts of the upcoming lessons. These lessons should be accessible for all learners and at the lowest level of Bloom's taxonomy (MAA, 2018). This leaves the tasks associated with a higher cognitive demand to be implemented in the classroom when the support of the instructors is present for struggling students (Moore, Gillett, & Steele, 2014). By implementing higher cognitive demand tasks, students are more likely to learn more mathematics (Boston & Smith, 2009). In addition, Moore, Gillett & Steele (2014, p. 423) report that students took "increased ownership of their learning" in the flipped classroom.

2.5.4 Metacognitive instruction

Students enrolled in developmental mathematics struggle with methods of learning mathematics, math anxiety, and studying for exams. Students will continue to struggle in these areas if they are not explicitly shown how to learn mathematics (Kiewra, 2002). Students can be shown how to take effective notes and taught study practices so that they are better prepared for summative assessments.

In addition, using formative assessments as a learning technique increases student learning (Boaler, 2015; MAA, 2018). Students are able to analyze their current learning and make adjustments before high stakes summative assessment. The feedback loop for formative assessments should be quick (MAA, 2018) and provide encouraging feedback for students to learn from (Boaler, 2015).

2.5.5 Conclusion

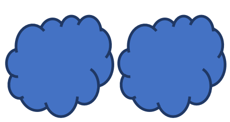
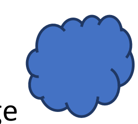
Active-learning classrooms enable students to be part of the learning process. These classrooms shift the learning process from procedures to understanding. In an active-learning classroom, students rely on each other to help make sense of the mathematics being taught. The instructor is a facilitator of the learning, instead of a giver of knowledge. One way to free up time to have an active-learning classroom is to flip the classroom so students watch videos outside of class on the basic concepts, and in class they begin to develop a conceptual understanding through activities. I will discuss some of the practices and activities that can be used to promote conceptual understanding.

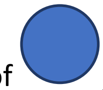
2.6 Conceptual Learning Best Practices

Learners of mathematics need to develop conceptual understanding and procedural fluency. Conceptual understanding is defined as “the comprehension and connection of concepts, operations, and relations” (National Council of Teachers of Mathematics, 2014, p. 7). Procedural fluency develops from conceptual understanding (Rittle-Johnson & Alibali, 1999) and is defined as “the meaningful and flexible use of procedures to solve problems” (National Council of Teachers of Mathematics, 2014, p. 7). Stigler, Givven, and Thompson (2009) found that students in developmental mathematics rely on procedures to solve problems and these procedures are often misapplied. They have not developed the conceptual understanding that is needed to facilitate procedural fluency. Many times “students do not view mathematics as a system because their teachers do not capitalize on opportunities to draw connections between mathematical representations” (Richland et al., 2012, p. 193). Thus, a student seems to think there is a new procedure for every type of problem instead of seeing the similarities between each type. Therefore, it is important for developmental students to establish a conceptual understanding so they can understand the procedures they have learned in their K-12 careers. “Students who develop an understanding of how concepts can be represented in different, connected ways will be able to link these representations to the procedures that are necessary to solve a problem” (Hodara, 2011, p. 13).

Developmental mathematics students who are referred to basic math classes may struggle with many different topics from basic mathematics. According to Stigler, et al., (2009), two of the concepts that students struggle with the most are fractions and their operations, and integer operations. I will highlight some of the best practices for teaching fractions and integer operations from K-12. I will focus on these two areas in depth because they are a foundation for the mathematical structures in algebra.

2.6.1 Conceptualizing fractions

Fractions represent one area where developmental mathematics students struggle. "Studies have concluded that students find fraction concepts to be much more difficult than whole number concepts" (Spangler, 2011, p. 13). Mathematicians realize they are an extension of the whole numbers and therefore exhibit most of the same properties of whole numbers. This logical extension does not translate into conceptual understanding for many students. Austin (2018) explains the disconnect in his letter to the editor of *Mathematics in School.* He explains that a student considers "two cats, two balls, two sweets and obtains the mental image that two is  – a fuzzy dreamlike image  which allows for generality in that two is not associated uniquely with any particular object yet which has concrete grounding in that in any particular situation they become actual objects" (p. 40). Thus, the student views the number two as two iterations of the same object. Then, with whole number objects, students can obtain a concrete example or a mental image of the arithmetic operations readily.

Conversely, when considering fractions, a student looks at “$\frac{1}{3}$ of , $\frac{1}{3}$ of , $\frac{1}{3}$ of , and obtains , , ” (Austin, 2018, p. 40). All of these are different compared to whole numbers. One-third is an operation on an object, not a discrete object that can be manipulated. Thus, it is no wonder that students have a challenging time performing operations with fractions when taken out of the context of the real world. In addition to the challenge of visualizing fractions as discrete objects that can be manipulated, comparing fractions is also challenging (Spangler, 2011). When comparing whole numbers and integers, inspection can find the number. “But with fractions, the notion of ‘next’ number does not apply (because between any two fractions there is always another fraction)” (p. 13).

Therefore, students who have already been taught fraction concepts and have misconceptions need to be taught using multiple representations. The troubles students have learning fractions “are related in part to teaching practices that emphasize syntactic knowledge (rules) over semantic knowledge (meaning) and discourage children from spontaneous attempts to make sense of rational numbers” (Cramer et al., 2018, p. 112). To help with students’ understanding, instruction “requires that several possible representations be available to allow a choice of those most useful for solving a particular problem” (Kilpatrick, Swafford, & Bradford, 2001, p. 102).

Part-whole, parts of a collection, number line, quotient of integers, and algorithms are five different approaches Spangler discusses in this book *Strategies for Teaching Fractions* (Spangler, 2011). He states “although there are pros and cons to using each model – and some are more difficult than others for students to comprehend… - all students should gain experience with each model" (p. 14).

2.6.1.1 Part-whole approach. In this approach. students "use spatial skills to see how a fraction is related to the unit whole" (Spangler, 2011, p. 15). Students use figures and can then compare and find equivalent fractions. When using this model, students have difficulties representing and modeling fractions greater than one (Izsák, Tillema, & Tunc-Pekkan, 2008). Therefore, when using this instruction, specific attention should be given to fractions with a value of more than one.

2.6.1.2 Parts of a collection. This approach has many real-world connections where one can consider parts of a set (Spangler, 2011). Having instruction in this method will allow teachers to use manipulatives to represent the different concepts. "Although time spent developing concepts through the use of manipulative and diagrams may be greater than the time needed to use a more traditional approach, less time is generally needed later for review and reteaching" (Spangler, 2011, p. 6).

2.6.1.3 Number line. This model provides an understanding of rational numbers as an extension of integers. It will also enable students to connect fractions to measurement (Spangler, 2011). Students can visualize equivalent fractions because the number line can be broken into successive partitions.

2.6.1.4 Quotient of integers. This approach calls for students to understand that $\frac{2}{3}$ is "2 divided by 3." This is not a common approach to teaching fractions and many times the division is done without students having a conceptual understanding of this approach (Spangler, 2011). Once students do start to understand this approach, they can begin making connections to decimals and percentages. In addition, students can start comparing decimal numbers, fractions and percentages. This type of approach has direct connections to fair division problems – for example, how do you divide seven cookies between four children?

2.6.1.5 Algorithms. Algorithms are important for procedural fluency of fraction operations. Procedural fluency should be accomplished by students having a conceptual understanding of the ideas first (National Council of Teachers of Mathematics, 2014). Thus, "the use of concrete objects that fit well with steps in algorithms appeared to help achievement, and this was more pronounced in retention" (Driscoll, 1984 as cited in Spangler, 2011, p. 16).

2.6.2 Conceptualizing addition and subtraction with integers

"Negative numbers" has at least three different meanings, depending on how the phrase is used: Unary, binary, and symmetric (Bofferding, 2009; Vlassis, 2008). Vlassis (2008) states the unary operation of the minus sign states negative numbers are a new class of numbers situated to the left of zero. For example, the unary meaning of a negative number is what is commonly known as a negative number: $-4, -9.3, -\frac{7}{5}$, etc. The binary meaning is the operation that changes the meaning of addition and subtraction. For example, $6 - (-5) = 6 + 5$ or $7 - 3 = 7 + (-3)$. The symmetric operation means that one takes the opposite of the number. For example, $-(-4) = 4$. Given all these different meanings, it is clear why students get confused when working with integers and their operations if they do not have a conceptual understanding of negative numbers.

While negative numbers have uses in algebra and beyond, it is very hard to construct real-world examples that use signed numbers (Peled & Carraher, 2007). For example, in this problem, a student was in debt by $400. She decreased her debt by $200. What does she now owe? As a mathematics instructor, I am looking for a student to write the expression

$$-400 - (-200) = -200$$

since debt would equate to a negative number. Many students will write

$$400 - 200 = 200$$

and then infer that it is still a debt from the context of the problem. While this is acceptable when solving application problems because students can infer the meaning from the problem, students have difficulty understanding what to do when given an algebraic expression without context (Peled & Carraher, 2007).

There have been numerous research articles on what types of manipulatives help students increase their conceptual understanding. I will discuss the types of manipulatives that are currently in use to help increase the conceptual understanding and what types work better than others. There is some limited research on the differences between virtual and concrete manipulatives, therefore I will discuss how best to use all the tools available.

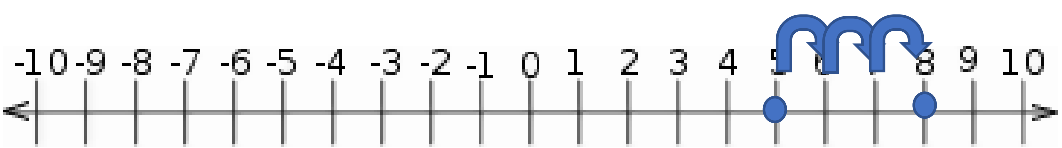
Negative numbers are a new class of numbers when students first start learning them. Using manipulatives can help students understand the different meanings underlying the new class of numbers. According to Sfard (1991), when students see the mathematical objects they become real, thus the objects exist and can be manipulated. Using manipulatives can help understand the three different meanings negative numbers have by the three different ways the objects are manipulated. In addition, using manipulatives gives students multiple representations of the same concept. According to Moreno & Mayer (1999), having students use multiple representations of the same concept enhances their understanding. Finally, writing down what they are discovering by using the manipulatives enables them to understand the symbolic representations.

2.6.2.1 Varieties of manipulatives. Manipulatives offer one way for students to gain a conceptual understanding of negative numbers. Here are a few different methods of using manipulatives for integer instruction.

2.6.2.1.1 Number line model. The number line model focuses on how much integers move when operations are done on them (Bolyard & Moyer-Packenham, 2012). The students must understand how addition and subtraction are associated with moving on a number line. For instance, if a student is presented with the problem $5 + 3$, the student must know that they start at the value of 5 on the number line and move 3 places to the right (see Figure 1).

Figure 1

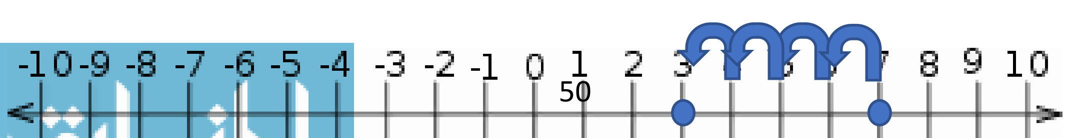
Number Line Addition



When students are subtracting, they must know they are moving to the left instead of right. For instance, if a student is presented with the problem $7 - 4$, the student must know that they start at the value of 7 on the number line and move 4 places to the left (see Figure 2).

Figure 2

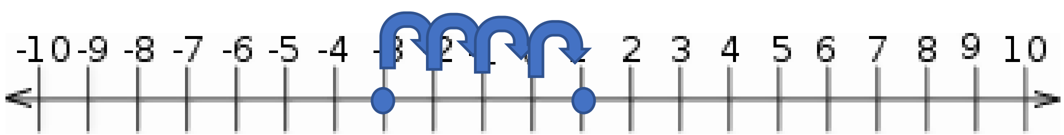
Number Line Subtraction



When negative numbers are introduced, the operation changes, based on whether the number is positive or negative. This is an example of the binary meaning of negative numbers. For instance, if a student is presented with the problem $-3 - (-4)$, the student must know that they start at the value of -3 on the number line and move 4 places to the **right** (see Figure 3). The move is to the right because, according to the binary meaning of the negative, the sign changes the operation from subtraction to addition.

Figure 3

Number Line Subtraction of a Negative



All these different procedures can be very hard for students to remember. Therefore, using other methods of instruction can enhance student understanding.

2.6.2.1.2 Debt/assets model in conjunction with a vertical number line. The debt and assets model focuses on how many places a number is from a starting point, not necessarily zero (Bolyard & Moyer-Packenham, 2012). For example, to help students understand problems that include subtracting negatives

$$5 - (-10)$$

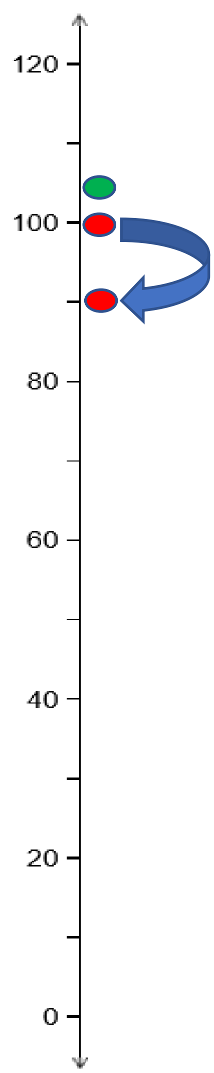
Stephan and Akyuz (2012) had students start with a person that had assets of \$105 and debts of \$100. This person has a net worth of \$5 but students are still able to take away \$10 of the debt. Since the student is taking away (subtracting) \$10 debt (negative number), this problem models

$$5 - (-10)$$

Looking at the vertical number line at the side (see Figure 4), if the 100 is replaced with zero, then the student can start to see how subtracting a negative number uses the binary meaning of negatives and changes to adding. This aids the students to proceed to determine conceptually what operations with integers mean (Stephan & Akyuz, 2012).

Figure 4

Vertical Number Line



2.6.2.1.3 Double abacus model (1). In one study done by Linchevski & Williams (1999), students used a double abacus and yellow and blue cards with numbers on them. The game was set up so groups of students were all doorkeepers for a party. They had to keep track of how many people went into the party (blue cards) and how many people left the party (yellow cards) on the double abacus. One side of the abacus represented students coming to the party (positive numbers) and one side represented people leaving the party (negative numbers). Students kept track of the number of people at the party on a double abacus. Periodically, each doorkeeper reported to the gate controller to make sure there were not too many people at the party. Students were able to understand they can take off or add the same number of beads on both wires, or take from the opposite color instead of adding to the color picked in the case they did not have the correct amount of beads in the color they needed (Linchevski & Williams, 1999).

2.6.2.1.4 Double abacus model (2). In another study done by Linchevski & Williams (1999), students again used a double abacus, but this time they also used dice. The students would throw a blue and yellow die and score similar to the door game where blue was positive and yellow was negative. The students were encouraged to be fair and therefore started scoring only the difference of the two dice, whichever was more. When the students mastered the idea, a die with addition and subtraction symbols was included. Next, after mastering the addition and subtraction concepts, dice that had $+3, +2, +1, -1, -2, -3$ on the faces replaced the blue and yellow dice. Finally, the addition and subtraction were re-introduced. Through all the dice games, the students were encouraged to say and write the mathematics symbolically (Linchevski & Williams, 1999). Both studies found that students were better able to conceptualize integers and their operations. The party version helped students internalize the process of integer addition and subtraction and may be best when the integer concept is first met. The dice game helped students see integers as objects that can be manipulated. Neither of the double abacus methods of manipulatives leads to using the same type of manipulative for multiplication. This may lead to some confusion when integer multiplication is introduced. Some students struggle to see the similarities and differences when different methods are used for concepts that build off each other as multiplication is built from repeated addition.

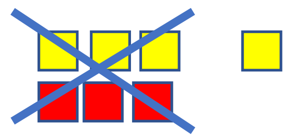
2.6.1.2.5 Algebra tiles/flip counters model. Bolyand and Moyer-Packenham (2012) suggest the following methods to use algebra tiles to enhance students understanding of integer addition and subtraction. The students use a double-sided algebra tile or flip counters. The yellow side represents positive numbers and the red side represents negative numbers. When using algebra tiles, different size tiles can represent different objects including variables. For instance, many sets of algebra tiles contain a small square, a rectangle, and a large square. The dimensions of the rectangle are the same as one side of the small square as the short side and the same as one side of the large square as the longer side. The tiles are then typically defined as "1" for the small square, "x" for the rectangle, and "x^2" for the large square, but the tiles can be defined differently for other problems.

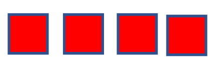
Students work with the algebra tiles by first representing given values and translating between the visual model and the written mathematics. Students are introduced to two fundamental concepts – only objects of the same size can be combined, and two objects that are the same size and opposite colors are zero pairs. For example, if a student has two small red squares and three small red squares, they can combine to make five small red squares. Written mathematically, this would be $-2 + (-3) = -5$. If a student has two small red squares and three red rectangles, they cannot combine because they are different size objects. Written mathematically, this would be $-2 + (-3x)$. This helps reinforce the concept that integers and terms with variables are not like terms and therefore cannot be combined together. The concept of zero pairs enables students to eliminate tiles when there is one positive and one negative tile of the same size. Students are also able to *add zero* to an expression to enable some manipulation that may be needed. Examples of integer operations using algebra tiles:

a. $4 + (-3)$ 

Three of the yellow tiles combine with the three red tiles to form three zero pairs and thus get eliminated. Therefore, one yellow tile is not eliminated and the result of

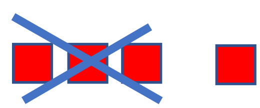
$$4 + (-3) = 1$$

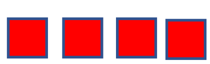


b. $-4 - (-3)$ 

One of the ways subtraction is defined is *take-away*. Thus, for the current problem, the student starts with four red squares and then needs to *take away* three red squares. This results in one red square left over.

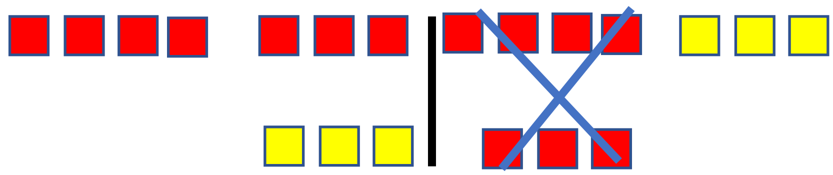
$$-4 - (-3) = -1$$



c. $-4 - (-7)$ 

This case is usually the most challenging for students. Similar to the previous problem, the student starts with four red squares. The student needs to take away seven red squares, though. Since there are not enough red squares to take away, the student must add three zero pairs to the expression so that there is a total of seven red squares to take away. After the seven red tiles are taken away, there are three yellow tiles left over.

$$-4-(-7)=-4-(-7)+\big(3+(-3)\big)=-7-(-7)+3=3$$



One benefit of using algebra tiles is that they can also be used to enhance understanding of multiplication, multiplication of algebraic expressions, factoring, and completing the square.

2.6.2.1.6 Conclusion. The research suggests that using any type of multiple representation helps students gain a better conceptual understanding (Bolyard & Moyer-Packenham, 2012; Moreno & Mayer, R., 1999; Stephan & Akyuz, 2012). Some methods help more than others (Sherzer, 1973 as cited in Bolyard and Moyer-Packenham 2012, p. 94). Linchevski & Williams (1999) found that students had trouble moving from negative to positive or positive to negative when using the double abacus model (1) because subtraction was modeled as undoing instead of taking away. Conversely, the double abacus model (2) increased students' conceptual understanding of integer operations because the dice game used positive and negative integers and students had to write out the mathematics of the procedures they were using. In the study of virtual algebra tiles, Bolyard and Moyer-Packenham (2012) found that students were able to translate from pictures to written/metaphorical and back quite well but struggled translating to and from symbol representation. The goal of all these methods is to increase students' understanding. Many of the studies focus on only one method of using manipulatives. It may be beneficial to see if employing multiple methods of instruction will help students' understanding or hinder their understanding.

2.7 Summary

Many different models of developmental mathematics education are being tried now. Many of these models change how often the course meets and may have in-time remediation for the topics that students struggle with the most. The literature suggests that shortening the time in developmental education leaves fewer exit points for students and therefore increases student progression through the sequence.

Most of these models do not change how the course content is delivered. This leaves the students needing multiple levels of developmental mathematics at a continued disadvantage because they have been exposed to all the material in their previous school experiences and did not learn it then. If material is presented in the same fashion, students will struggle a second time to learn it. Active learning helps students make sense of the content on their own and together with their instructors and fellow students (CBMS, 2016; MAA, 2018; National Council of Teachers of Mathematics, 2014; National Research Council, 2003). Thus, students are better able to learn the mathematics they are struggling with and correct any misconceptions they may have (Hammerman & Goldberg, 2003).

The students who are referred to multiple levels of developmental mathematics face challenges other than their depth of mathematics knowledge. They are also usually at-risk students who may have to work part-time or full-time outside of school. They may be single mothers or first-generation college students. Thus, they have many responsibilities outside of their education that may take precedence. In addition, these students struggle with how to learn math, math anxiety, and test-taking. The bright side is that studies show that helping students adjust to college life and learn study skills benefits their progression through the developmental sequence.

All of these studies look at one aspect of developmental mathematics and adjust it. There is a gap in the research about putting many of the best aspects together to determine if a whole package will benefit developmental students referred to multiple levels so they can progress to their degrees at a similar rate as students who are referred to fewer levels of developmental mathematics or even referred to credit-bearing mathematics. "By teaching math in an integrated fashion, teachers will actually save time in the long run. They will eliminate the need to go over the same content time and again because students did not learn it well in the first place" (National Research Council, 2003, p. 22). Therefore, if the course is taught with a focus on student understanding and an accelerated format not only will students gain the understanding, but they will also have a similar time to degree as students that are not referred to multiple levels of developmental mathematics. It is important to set all students up for success by incorporating an accelerated course with active learning, placing an emphasis on conceptual understanding and offering support with study skills and test-taking strategies.

Chapter 3 Methods

3.1 Introduction

There is a nationwide push to reform developmental mathematics education. In fact, many states have enacted laws regarding reform in developmental mathematics education. According to Baily and Jaggers (2013), "the current systems of developmental education needs improvement" (p. 18). Many of the studies that have been done look at the current state of developmental mathematics education (Bailey et al., 2013; Bailey & Jeong, 2010; Bonham & Boylan, 2011; Fong et al., 2014), not at holistic interventions that help improve students' mathematical understanding. There are studies that look at how providing corequisite remediation (Bracco et al., 2015; Brothen & Wambach, 2004; Rutschow & Schneider, 2011) or accelerating coursework (Bailey & Jeong, 2010; Cafarella, 2016; Fong et al., 2014) affects students' progression through the developmental and first college credit-bearing mathematics course. These studies show that students being referred to one developmental mathematics course can complete their credit-bearing course in one semester if they enroll in an accelerated or corequisite class. These same studies also show there are some gains for students who are referred to two or more developmental courses but the number of students passing a credit-bearing math class is still quite low among those who must take multiple developmental courses.

In addition, there is evidence that changing the methods of teaching from a lecture format to a flipped-classroom format (MAA, 2018) or an active-learning classroom (Bonham & Boylan, 2011; CBMS, 2016; MAA, 2018; National Council of Teachers of Mathematics, 2014; Ngo & Kosiewicz, 2017) can increase student understanding and progression through the developmental sequence. Furthermore, many studies have been done on students' mindsets (Boaler, 2015; Dweck, 2008) and how students persevere in the face of difficulties (McKay, 2015). Finally, there are best practices for teaching fractions (Izsák et al., 2018, 2008; Spangler, 2011), and integers (Hunt, Nipper, & Nash, 2011; Linchevski & Williams, 1999; Peled & Carraher, 2007; Vlassis, 2008) that support the development of conceptual understanding. Much of the research in developmental education currently focuses on the structure of the course, such as corequisite model, emporium model, accelerating the coursework, etc. There is a gap in the literature in regard to changing the design features of a course, such as active learning, using pedagogy from K-12 to enable students to gain a conceptual understanding and addressing the gaps and misconceptions and incorporating features that support students' mindset and study skills. My study attempts to formulate four design principles that are focused on student learning and then incorporate design features to support these design principles and provide a holistic learner-centered course design.

This study is a design experiment that uses the best practices found in the research for adult education and K-12 education, classroom environment, and content learning to construct a course intended to maximize student learning in developmental mathematics. This chapter will discuss the context and population under study. It will then discuss the research problem, research questions, and the research design. Finally, the chapter will conclude with the limitations, delimitations of the study and my reflexivity statement.

3.2 Study Context

The study took place at a large midwestern urban university. I am the course coordinator for a course that is focused on students with the most severe mathematics difficulties at this university. A student who is referred into this course has a Math ACT of 16 or lower or a Wisconsin Mathematics Placement Exam score of zero. As discussed in the literature review, these students have a fragmented understanding of basic number sense, solving arithmetic problems involving rational numbers and percentages, modeling, integer operations, fraction operations, exponent properties, polynomial operations, solving linear equations and inequalities, and anything based on these concepts. Most of the students in this course have graduated from a Wisconsin high school. According to the Wisconsin Department of Public Instruction, students need a minimum of 3 credits of mathematics for graduation starting the 2016-2017 school year ("Graduation Requirements WI DPI," 2019). Thus, students who are graduating from a Wisconsin public school typically have had Algebra I, Geometry, and Algebra II, or Integrated Math I, II, and III, yet their ACT math scores show a limited understanding of the material in Algebra I and below. The course at this university is designed to be a combination of basic mathematics and beginning algebra. There are 7 sections that run in the fall semester and about 3 sections that run in the spring semester. The sections have an initial cap of 22 students per section and max out at 25 students per section. The course is a six-credit-hour course, but all the credits are developmental credits and therefore do not count toward graduation. The students meet four days a week for 75 minutes each class. The course takes on the structure of an accelerated course (Cafarella, 2016; Jaggars, S. S., Hodara, M., Cho, S., & Xu, 2015; Sheldon & Durdella, 2010) because it meets twice as often as a typical three-credit course. It is not considered a corequisite course (Bracco et al., 2015; Brothen & Wambach, 2004) as it does not have students taking a credit-bearing class in conjunction with their developmental coursework. The students also have a required 75-minute discussion section that meets once a week where they practice their study skills (notetaking, test-taking strategies, etc.) and work on their online homework. This additional support is similar to an additional success course (Brock, 2010; Rutschow & Schneider, 2011) and is designed specifically for the students in this course.

Experienced adjunct lecturers and graduate teaching assistants teach the course. The instructors are assigned to teach this course based on their teaching abilities and their desire to teach in a student-centered environment. At the beginning of the semester, there is an intensive two-day training program for the instructors to learn how to teach in an active-learning environment. In addition, there are meetings every two weeks to discuss upcoming learning outcomes and activities, students' difficulties, and speakers from around campus to discuss resources for at-risk students.

The students in this course have all been exposed to the mathematical content. Unfortunately, they have developed a fragmented understanding of the material and many have a challenged history with mathematics. This course is designed in such a way that it enables the students to develop the connections needed to make sense of the mathematics. In addition, the students are given the opportunity for success when they struggle with the material through growth mindset messaging. The aim of this study is to enable all students to build the foundation they need for success in any mathematics they take in the future.

3.3 Population

The population for this study is students who have the most severe mathematics history and have been referred to two levels of developmental mathematics at a large midwestern university. These are students who have an ACT math score of 16 or less or a Placement Exam level 0 because that is the placement level into this course. The study includes all sections of this course that are running in Fall of 2019. The sample is made up of the students who agreed to be part of the study. All students in all sections had the same experience, whether they consented to be part of the study or not. The students who did consent to be part of the study did not receive any incentives for doing so.

I met with each section of the 7 sections the first day of class, described the study, and asked students to provide consent to be part of the study. While the students were filling out the consent forms I waited in the room. In the class that I taught, I had a colleague ask for consent from the students. All students were informed that providing or not providing consent would in no way affect their standing in the class or at the university.

There were a total of 93 students that consented to be part of the study, and 162 officially enrolled in the course. Of the students that consented to be part of the study, there were 62 females, 29 males and 2 that did not answer the gender question. There were 57 students that identified as White, 16 Hispanic, 19 Black, 3 Asian, 8 Multicultural, and 6 that did not answer (see Table 1 for comparison data). As a proportion of the population, there were more underrepresented minorities (non-white) in the sample (39%) than there were in the population of students (19%) that took the course. I am only able to compare this category as it appears that students were able to check multiple race categories in the population data.

Table 1

Demographic data for study group

Race	Number of students in the study Fall 2019	Number of students in the course Fall 2019
White	57	132

Hispanic	16	14
Black	19	62
Asian	3	8
Multicultural	8	68
American Indian		2
International		12
Did not answer	6	

All students scored a zero on their placement test. A zero on the placement test indicates they are referred to multiple developmental courses before they are able to enter their gateway mathematics course. The ACT Math scores ranged from 14 to 23 (see Table 2).

Table 2

ACT Math Scores

ACT Math Score	*Number of Students with Score*
14	2
15	13
16	22
17	22
18	9
19	6
20	2
Above 20	3

The students in the study had an average of 3.1 years of high school math.

At the end of the semester, there were 15 students that quit coming to class that had enrolled in the study. They either withdrew mid-semester or simply quit coming to class. These students did not complete the post-ATMI survey and the end of the semester Class Impression survey. I contacted these students multiple times to request their responses despite their lack of attendance in the class at the end of the semester. None of the students responded. Thus, the survey responses at the end of the semester are missing student responses.

3.4 Research Problem

Students who are placed in developmental mathematics at college have taken similar math classes at least once before in high school and many have negative histories in their mathematics education. While taking math in high school (or before), they developed fragmented understanding and a reliance on procedures that prevent them from succeeding in a college-level credit-bearing course. Typical developmental mathematics classrooms do not address the fragmented understanding through an emphasis on conceptual understanding. Rather, they teach the material in the same way the students were taught in high school, only faster and more self-directed. These typical developmental mathematics classrooms usually provide little to no support to help students overcome their history in mathematics and provide little instruction on how to be a mathematics student.

3.5 Research Questions

1. How do the design changes affect student learning? In particular:
    a. How does targeted conceptual understanding instruction affect a student's conceptual understanding of and procedural fluency in integers and fractions and their operations?
    b. How do the design changes of the course compare success rates of current students to historic data from students before the reform took place?

2. How do the design changes affect students' attitude toward mathematics, and is there a change in discourse habits over the course of the semester?
3. What are the students' perceptions of the course design? In particular:
    a. What features of the course design do students believe is most important for their learning?
    b. What are the students' perceptions of learner-centered assessment system?
    c. What are students' perceptions of the learning opportunities designed to promote a growth mindset?
    d. What are students' perceptions of learning opportunities to promote a conceptual understanding of integers and fractions and their operations?

3.6 Research Design

This study was conducted using a design-based experiment. Anderson & Shattuck (2012, p. 16) explain that a design-based experiment "begins with an accurate assessment of the local context; is informed by relevant literature, theory, and practice from other contexts; and is designed specifically to overcome some problem or create an improvement in local practice." As stated before, many best practices have been researched in mathematics education in general and in developmental mathematics education specifically. Many of the interventions are studied as individual interventions and not studied in conjunction with other interventions. This study was designed to examine the entire course setup holistically, the "interventions as enacted through the interactions between material, teachers, and learners" (Collective, 2003, p. 5). Using the design-based experiment allowed me the opportunity to "implement a theoretically grounded instructional approach and to test and refine the theory guiding that implementation" (Moss & Haertel, 2016, p. 166). Consequently, as Cobb, Zhao & Dean (2009, p. 308) state, "theory is seen to emerge from practice and to feed back to guide it." According to Steffe & Thompson (Steffe & Thompson, 2000), "hypothesis formulation, experimental testing, and reconstruction of the hypothesis form a recursive cycle."

The construction of this study as a design-based experiment was multifaceted. My purpose was to produce a course that provides students the ability to succeed in their developmental mathematics course and provides them with the tools they will need to succeed in their subsequent mathematics course(s). Thus, to test only one or two instructional interventions and determine their effect using an experimental design would not inform how to combine best practice interventions. "A basic assumption of the emergent perspective [design-based experiments] is, therefore, that neither individual students' activities nor classroom mathematical practices can be accounted for adequately except in relation to the other" (Paul Cobb et al., 2009, p. 310). The interventions are all grounded in theory and therefore have been shown to have a positive effect on their own. As Cobb, et al. (2003, p. 10), state "design studies are typically test-beds for innovation." This study was testing the effectiveness of many different innovations working together. By looking at many aspects of a single class design, it is possible to start developing a general theory to better serve developmental mathematics students. "It is this quest for generalizability that distinguishes analyses whose primary goal is to assess a particular instructional innovation from those whose goal is the development of theory that can feed forward to guide future research and the development of activities" (Paul Cobb et al., 2009, pp. 327–328).

When analyzing the data from this study, I used a concurrent mixed-methods approach (Creswell & Creswell, 2018). All students in every section were given a pre-test at the beginning of the semester and a post-test at the end of the semester whether or not they are part of the study (See Appendix C). This test focused on knowledge of fractions, integers, and their operations because these topics form a foundation for further mathematics learning and the population under study struggles with these concepts. The pre-test was constructed using a blueprint based on the Common Core State Standards - Mathematics (CCSS-M) standards of Number and Operations – Fractions (third through fifth grade) and Number System (sixth through eighth grade). These standards were chosen because they are standards adopted by Wisconsin Public Schools and they are the content areas under study. The test blueprint and the test items were given to two mathematics instructors to examine for validity. Feedback from these instructors was used to add, delete, and modify items. The test items also appeared on the final exam for the course to provide a post-test without taking additional class time.

Students took the Attitude Toward Mathematics Index (ATMI) (Tapia, 1996) (see Appendix D) as a pre-survey at the beginning of the semester and a post-survey at the end of the semester. The use of the survey will help determine if student's attitudes toward mathematics have changed after completing the course. This survey has 40 items based on value, anxiety, motivation, confidence, and enjoyment. Each item is scored using a Likert scale where 1 is "strongly disagree" to 5 being "strongly agree." ATMI has a coefficient alpha value of .9667, which indicates good reliability. To establish content validity, the original instrument started with a blueprint of the domains being assessed that related to the four variables measured. In addition, the items were examined by two experienced mathematics teachers.

Some items were added and modified based on feedback from these teachers. Construct validity was achieved by showing item homogeneity. The item-to-total correlation of 0.49 indicated the instrument is unidimensional. Tapia also performed principal component analysis with a varimax rotation (Tapia, 1996). The factor structures were selected based on eigenvalues and a scree plot. The factors with eigenvalues greater than one were selected. The four-factor structure provided the best simple fit and accounts for 59.22% of the variance. Since there were six original variables and only four factors, the structure was examined (Tapia, 1996). It was determined that two variables combined into one factor and one variable was not relevant from the factor structure. In fact, this was the variable that was associated with the nine dropped items. The four factors in the final model were a sense of security (reliability of 0.95), value (reliability of 0.86), motivation (reliability of 0.89), and enjoyment (reliability of 0.88).

The qualitative portion of this study was multi-dimensional. First, most sections had a classroom observation conducted to gauge the level of mathematical discourse occurring in the class. The observations were transcribed and compared with observation notes to determine the amount of discourse that was occurring in the classroom. There were two student focus groups conducted. The first had two students and was conducted at the end of the semester. The second had three students and was conducted the beginning of the spring semester. The purpose of the focus groups was to find student's perceptions of classroom connectedness, growth mindset activities, the learner-centered course assessment system, and class learning activities. Finally, there was a quick student survey to find students' perceptions of the design features of the course approximately three-fourths of the way through the course. See Table 3 for a description of all the research questions and their methods of analysis.

Table 3

Research Question Summary and the Methods of Analysis

Research Question	***Analysis Method (See Appendices for Instruments)***
Q1: How do these design changes affect student learning?	
Q1a: How is students' conceptual understanding and procedural fluency affected?	Pre- and post-tests (Appendix C for pre-test)
Q1b: How do the students' pass rates compare to historical data?	Overall pass rates for the class will be compared to 2012 pass rates. 2012 was the last year this class was taught in a traditional format
Q2: Changes in student's attitudes after growth mindset intervention	ATMI (Appendix D)
Q2a: Changes in discourse over the semester	Observation and videotaped data, and focus group data
Q3: What are student's perceptions toward the course design?	Focus group (Appendix A for interview protocol) Student Survey (Appendix B)

3.7 Data Collection

At the beginning of the semester, students were given a pre-test (see Appendix C) to determine their understanding of integers and fractions and their operations. Integers and fractions were chosen because they are areas where students referred to developmental mathematics struggle (Stigler et al., 2009) and the content provides a foundation for further work in mathematics. In addition, students completed the Attitude Toward Mathematics Survey (Tapia, 1996) (see Appendix D) the first week of classes.

Throughout the semester, I performed classroom observations. I choose multiple instructors. Each instructor had an observation to look for evidence of discourse in the classroom environment. The classroom observations were done the third week of classes as classroom norms had been set and it was the beginning of the instruction on fractions and number lines.

About three-fourths of the way through the semester, students were given a student survey (see Appendix B) that focuses on their perception of learning activities that are used in the course.

At the end of the semester the students participating in the study and not in my class were asked to participate in a focus group. Two students volunteered and we met and talked about their perceptions of the course. At the beginning of the following semester, I sent out another email to all the students, including my own students, and asked them to be part of a focus group. Three other students volunteered for this focus group. Both focus groups were asked the same questions.

At the end of the semester, students had questions similar to the pre-test on their final exam to determine their understanding of integers, fractions, and their operations. They were also asked to fill out ATMI again. The administration of these tests and surveys tracked the changes in student attitudes and understanding throughout the semester.

At the conclusion of the semester, I collected historic data for the courses that formally covered the same material that is taught in the course. I compared the data from this semester to the historic data to determine if there is a change in pass rates.

3.8 Analysis

I performed a concurrent mixed-methods analysis of the data. At the beginning of the semester, after the initial pre-test and the ATMI were administered, I performed a statistical analysis of the sections using ANOVA to determine if there were differences in prior knowledge, demographics, or attitude toward mathematics between the sections. There was no difference in the different sections.

Throughout the semester, I examined student and instructional artifacts and videotaped classroom observations from all sections to determine if any themes emerge. I focused on student participation in classroom discourse and class activities that day and how that affects students' understanding. I observed classrooms on the days where the fraction and integer instruction were taking place. I used basic qualitative analysis to determine if any themes emerge.

About three-fourths of the way through the semester, I had the students complete a survey that focuses on their perceptions of the learning activities that are used in the course. The survey was quantitively analyzed to determine what students perceive as the most important aspect of their learning (see Appendix A). The short answer questions were qualitatively analyzed to determine if any themes emerge.

About three-fourths of the way through the semester, I conducted a focus group session with two students. In addition, I conducted a second focus group session with three students at the beginning of the following semester. I discussed with the students what aspects of the course design they believe are most important for their learning. I also discussed with them what they felt was least important for their learning. I analyzed these results using qualitative analysis to determine if there are emerging themes.

At the end of the semester, I analyzed the pre-test results and the corresponding questions that appeared on the final exam to determine if there is a statistically significant difference using a t-test. I also reported the pass rates and how they are associated with the demographics of the students.

Finally, I analyzed the pass rates of the students of the study semester as compared with students from Fall 2010-Spring 2011, Fall 2011-Spring 2012, and Fall 2012-Spring 2013. I included three years of data because there were a small number of students enrolled in the course each of the year. The comparison will be students who started in this course in fall semester and how they performed the subsequent spring semester.

3.9 Qualitative Analysis

The basic qualitative analysis was performed using Nvivo 12. After reading through the transcripts and short answer questions, I first coded the data based on the research questions.

Starting with question 2, I started nodes based on mentions of affective characteristics. These nodes were *Correcting Mistakes Perception, Exam Retakes Perception, Growth Mindset Instruction and Motivation.* As these items were mentioned evidence was placed into that node. After completing organizing the data into the nodes, I read through the evidence in each of the nodes to develop themes. For question 2, data was obtained from both the short answer responses and the focus group. Thus, I was analyzing what students said in regard to different aspects of each of the themes. For instance, in the node for *Correcting Mistakes* students responded one of the three most beneficial aspects of the course, “correcting mistakes because it allows you to go back and fix the mistakes that you made and helps you learn from them", "because I got to see my mistakes and learn from them", and "I would tell them the most beneficial parts of the class are the corrections we are allowed to make and the retakes n the exam we are given the chance to take." The students in the focus group talked about going to the board and being able to fix their mistakes as the class was working through problems. For example, when we can "take the time to go through and change them and everything, [that] was helpful too" and "yeah, I agree with going up there and, like, just everyone kind of fixing it together." I performed the same analysis for the other three nodes also.

Question 3 is students' perception of the different design features of the course. Thus, the nodes I used for question 3 were *Correcting Mistakes, Exam Retakes, Grading System, In-Class Activities, Mandatory Attendance, Mandatory Discussion, Online Homework, Order of the Topics, Study Skill Instructions, TED Talks, Warm-up Activities,* and *Watching Videos.* These nodes were coded using both responses from what is one of the three most beneficial aspects to your learning and what is one of the three lease beneficial aspects to your learning. Therefore, when I coded the data, I made notes if the evidence was ambiguous. For instance, in the node for *Mandatory Attendance,* one piece of evidence was "office hours and attending class". For this piece of evidence, I made a note that this came from answering the question what is one of the three most beneficial aspects to your learning. Many of the answers for these questions were short and had little description. Therefore, I used the frequency of the response and interpreted the more robust responses that were included. For example, in the *Mandatory Attendance* node there were 32 references to mandatory attendance as one of the most beneficial aspects to students' learning. Some of the responses consisted of "paying attention and attendance", "mandatory attendance because it is mandatory", and "mandatory attendance, because if you do not attend you do not pass the class." There were more robust responses that included, "I choose mandatory attendance because if you just blow off this class you will fall behind and then you struggle even more. I believe it is important to show up for a class like this because you are constantly learning new things and it's not easy to catch up if you miss a day."

The qualitative analysis in this study informed why students perceived different aspects of the course as beneficial to their learning or not beneficial to their learning. I was able to understand more of the different rankings of the design features of the course. This helped inform the analysis and discussion of these design features. For example, study skills are very important for students that are referred to multiple developmental courses. But students did not feel that study skills instruction was beneficial to their learning. Therefore, if study skills instruction is to be included in the course, the course coordinator or instructor should make the study skills instruction transparent for the students.

3.9 Limitations

This study is based at an urban access university with a specific population of students. Some of the practices may not work with students of a different population. Also, the instructors that teach this course are open to teaching in an active-learning classroom. The course may not be effective with instructors that are uncomfortable teaching in an active-learning classroom.

Additionally, the students self-selected to participate in the focus groups. Therefore, the size of the focus groups was limited, and the aspect of the data may not be a representative sample of the students in the study.

3.10 Delimitations

This study is not set up as an experimental design. Thus, it will not have a control group and a treatment group to compare to each other. There is a comparison group that was used from the fall of 2012 and the spring of 2013. This is the last semester the course was taught in a different format. The practices used in this study are all based on the literature and therefore have been shown to be effective by themselves. The purpose of the study is to determine if the various practices can work together and show effectiveness together.

3.11 Reflexivity

“Reflexive engagement while planning, conducting, and writing about research promotes an ongoing, recursive relationship between the research’s subjective responses and the intersubjective dynamics of the research process itself” (Probst, 2015, p. 37). Thus, according to Pillow (2003), “the focus requires the researcher to be critically conscious through personal accounting of how the research’s self-location (across for example gender, race, class, sexuality, ethnicity, nationality), position, and interests influence all stages of the research process.” (p. 178). As a researcher, one must identify one’s position to research participants and address the issues that this position inherently holds. A researcher can misuse reflexivity by simply being aware of the biases and issues of inequity and not doing anything to mitigate these issues and biases (Pillow, 2003).

Berger (2013) states there are three ways a researcher can impact the research. (1) Participants are more willing to share if they perceive the researcher to be sympathetic. (2) The research may shape the relationship based on their positions. (3) The researcher's background and worldview may shape how the research is performed (Berger, 2013). I also believe this to be the case. It is important, in my role as a researcher, to be aware of how my students and instructors react to me as either their instructor or supervisor. In addition, I need to be aware of how my past and present experiences shape who I am as a researcher, instructor, and supervisor.

Reflexivity needs to take place in all parts of a research process. "For example, during interviewing, being self-reflective helps the researcher to identify questions and content that he or she tends to emphasize or shy away from and to become aware of own reactions to interviews, thoughts, emotions, and their triggers. During content analysis and reporting, it helps in alerting oneself to 'unconscious editing' because of [one's] own sensitivities and thus enables fuller engagement with the data and more in-depth comprehensive analysis of it" (Berger, 2013, p. 3-4). For instance, if a person is secular, they may not notice the importance of someone speaking about God in their life. If they are not aware of the importance of the statement, they may come to a different conclusion than a researcher who is a practicing Christian.

Thus, analyzing my beliefs and past experiences, I believe in the equality of opportunity. I hope to embody the ideal that no person is discriminated against based on age, gender, race, disabilities, SES, or religious orientation. By this, I mean giving everyone the same opportunity to learn based on where they are at a given time. Thus, if a student has a physical disability, I need to work with that person in order that they learn to the best of their ability. I take this to mean more than accommodate the individual; I mean sitting down with the individual to ensure they can get the most out of their education. If one of my students is struggling financially, I do my best to help them get the materials they need until they can get the materials for themselves. This gives all students an equal opportunity to learn without worry about missing work.

Conversely, this also means that I hold students accountable for material they can complete. If a student is not completing work because they choose to go out over the weekend, make a choice that other schoolwork is more important, or they must work, they are held accountable for those actions. If the reason the student is not completing work is that they do not know how to schedule their school, work, or life in balance, this is something we talk about in class. When I was working on my bachelor's degree, I had to work full-time because I did not have enough money to pay bills. It was extremely hard for me to juggle my work and school to find a balance and I ended up dropping out of school without withdrawing from classes. I then choose to stay out of school and work until I was stable enough to work part-time and go to school. When I went back to school, I was more dedicated and ended up finishing my degree two semesters after I restarted. I understand the struggle some of my students face financially and hope that some aspects of this course can help them learn how to find the balance they need to succeed. That being said, I am a white woman. That being said, there are many other challenges my students have faced that I have not experienced. I still try to be supportive and empathic to all my students.

As a researcher (and instructor), I need to be aware that everyone sees himself or herself as belonging to different groups. Since I am not a part of many of the same groups as my students, I need to be aware of my position of power. I am not only their instructor; I am also judging some of their work as a researcher. I do try to mitigate this position of power by telling the students some of my personal story – including the story I told above. I hope that they can identify with me on some level and can feel comfortable in my classroom. I am also very aware that many of my students have personal difficulties outside of the classroom that I have never experienced. I try to be as supportive as possible while still holding them accountable for the choices they make.

In addition, I believe that all people are individuals and as such have their own worth as individuals. Every instructor and student bring their own background and beliefs with them to the classroom. With these varying backgrounds and beliefs, everyone has something to contribute to the class. As an instructor, I need to be able to make a safe space for my students to contribute to the class to enhance learning for everyone. As a researcher, I need to be aware of everyone's background and not judge his or her interviews on the words alone. I need to be aware they may have the knowledge of what they are doing but do not have the words to express that knowledge.

During my study, I needed to be aware of my position of power in the classroom and to my instructors. I needed to ensure that I try to mitigate this position as much as possible. In the classroom, I did this by being open to students and being aware of their positions within the classroom. I tried to make a safe space for students to be open to making mistakes and learning new ways of thinking about mathematics, which can make them feel uncomfortable. Regarding my instructors, I needed to reassure them that their performance on the new teaching methods would not determine their employment. During regular instructor meetings, I had some instructors get nervous when I am taking notes. The notes were reminders for me of items that I need to bring to attention.

3.12 Conclusion

This course is designed to meet students who have been referred to multiple developmental mathematics courses where they are and enable them to be successful at this course. It is also designed to give them the tools to use as they progress through their mathematics sequence on their road to graduation. The study used a design experiment approach in which multiple research-based interventions are combined. The study analyzed students' proficiency in integers, factions, and their operations. In addition, the study analyzed whether there is a change in a student's attitude toward mathematics and perceptions of the course design.

The next chapter will focus on the course design. Each design principle will be described and the research base for the principle is discussed. Then examples of how each of the design principles is implemented in the course are discussed. Finally, I will discuss how each of the design principles will be measured.

Chapter 4 Design Principles of the Course

The students who are referred to this course have a severe history in their mathematics education. This course is designed to help students overcome that history and proceed on the path toward success in mathematics and degree attainment. With that overarching goal at the forefront, the course has four main design principles:

1. *Create an equitable environment where students are comfortable participating in mathematical discourse.*

2. *Organize learning outcomes and class activities in a way that promotes conceptual understanding and procedural fluency among mathematical objects.*

3. *Provide a variety of learning opportunities about different affective characteristics to promote a positive attitude change in students' mathematical mindset.*

4. *Create an assessment system that explicitly values learning including cognitive and social-emotional outcomes.*

I will discuss each of these design principles, including how they are grounded in literature, how the principle is implemented into the course, and how each is measured for effectiveness.

4.1 Design Principle 1

Create an equitable environment where students are comfortable participating in mathematical discourse.

For students to learn, they need to feel safe. In a developmental classroom where many students have had negative experiences in mathematics classes, it is imperative to create an equitable environment for learning to happen. Zull (2002) says it best in *The Art of Changing the Brain*: "good thinking requires that we pay attention, but that is hard to do if someone threatens us" (p. 75). Therefore, "it becomes imperative to understand how to build positive social relationships that signal to the brain a sense of physical, psychological and social safety so that learning is possible" (Hammond, 2015, p. 45).

Creating an equitable environment involves teachers practicing culturally responsive teaching. Hammond (2015) states "neuroscience tells us the brain feels safest and relaxed when we are connected to others we trust to treat us well" (p. 73). Thus, the instructors should develop connections with their students at the beginning of the semester. Hammond (2015) describes some ways to build connections with students. First, "is to show we authentically care about who they are, what they have to say, and how they feel" (p. 75). Second, "we can build trust by being more authentic, vulnerable, and in sync with our students" (p. 79). One way to do this is to open yourself up to your students in an authentic manner. In addition, if the instructor is a student or researcher, Weimer (2013, p. 157) suggests the instructor "talk about what he is currently trying to master."

In addition, the instructors should facilitate connections among the students in the classroom. "Talking helps us process our learning. Talking helps us connect with others. Talking helps us expand our thinking when we hear the ideas of others" (Hammond, 2015, p. 148). Building a classroom around talking helps students gain a better understanding of the material being talked about. The classroom discussion helps uncover misconceptions and helps students see different viewpoints stemming from the same problem (Hammond, 2015; National Council of Teachers of Mathematics, 2014; Zull, 2002). One reason students are hesitant to talk in a developmental classroom is that they are afraid of making a mistake. "The challenge is that in most classrooms mistakes and errors are seen as something bad" (Hammond, 2015, p. 115).

Developing a classroom where mistakes are seen as learning experiences will foster discussions and connections among the students (Boaler, 2015; Hammond, 2015).

Finally, students need to be connected to the class as a whole. “The more time the teacher devotes to involving students in classroom climate issues, the more ownership they feel for the class and the more responsibility they assume for making the class a good place for learning” (Weimer, 2013, p. 158). As students feel the classroom is a good place for learning, they will want to attend class and participate.

In practice, this connection can be fostered even before the semester begins. One week before classes begin, an email is sent to the students describing the course and the expectations associated with the course (see Appendix F). Included in the email is a learner-centered syllabus (see Appendix J) and directions to respond to the email. The result of this response is multifaceted. First, the instructor knows that the student has an idea of what to expect before the semester begins. Second, many students ask questions about the course or the course materials and the instructor can respond to individual questions. Third, as the students respond, the instructor sends a quick response back letting students know that he or she is looking forward to having them in class. Finally, if students have not responded to the email, the instructor calls the student to let them know about the email. This initial contact with students helps them realize that their instructor cares about their success in the course.

The connections between teacher-student and student-student are further realized the first day of class, when there are multiple activities designed to help students familiarize themselves with each other and the instructor. One of these is People Bingo where students walk around and find students who fit the description on the Bingo sheet. They introduce themselves to each other; then when the class comes back together, each student introduces one of the students they met. After the introduction, the student says a little about themselves and the instructor makes a note about what the students shared. This helps the instructor remember the students' names faster and throughout the semester the instructor can bring some of this information into the course in the class activities or assessments. The first day of class concludes with a group activity through which students develop a set of shared course expectations. The instructor asks, "What types of expectations do you have for yourself, your instructor, and your fellow classmates this semester?" Then students work together to develop what they feel will be most beneficial to their learning.

At the beginning of each class period, there is a five- to ten-minute number talk. The number talks are designed to be accessible to all students, to be relevant to the upcoming learning outcomes, and have multiple paths to the answer or multiple answers. The first number talk we do is Ann, BRAD, Carol, DENNIS – What name comes next? There are multiple patterns that can be followed, and it is fun for students to think about what rules fit the pattern. This first number talk is low stakes because it does not have anything that students see as mathematical. Thus, students do not hesitate to give suggestions for the rules. Yet, when the talk is complete the instructor facilitates a discussion about pattern finding and how that relates to mathematics.

Then, throughout the semester, discussion is facilitated daily by the instructor. The discussions stem from workbook problems and, more importantly, mistakes on workbook problems, and class activities. The instructor asks open-ended questions to facilitate discussion. In addition, students are asked to expand on each other's explanations.

To understand how this design principle is realized in the classroom, students took an initial survey that measures a student's attitude toward mathematics. The students took the same survey at the end of the semester to determine if there was a change in their attitude toward mathematics. Also, I included artifacts of the initial contact with the students and the email exchanges that took place or notes on the phone conversations (see Appendix F). In addition, I had classroom observations that gave examples of different ways the instructors facilitate discussions in the classroom. Finally, at the end of the semester, I had two focus groups that discussed how students perceived their connectedness in the classroom.

4.2 Design Principle 2

Organize learning outcomes and class activities in a way that promotes conceptual understanding and procedural fluency among mathematical objects.

Students in developmental mathematics typically have a fragmented understanding of mathematics that has developed over their K-12 careers. This course attempts to promote conceptual understanding that, in turn, builds procedural fluency with the understanding that all students in developmental mathematics have some amount of prior knowledge of the course content. *Principles to Actions: Ensuring Mathematical Success for All* (2014, p. 7) describes the connection between conceptual understanding and procedural fluency as "conceptual understanding (i.e., the comprehension and connection of concepts, operations, and relations) establishes the foundation, and is necessary, for the developing procedural fluency (i.e., the meaningful and flexible use of procedures to solve problems." Therefore, the course design should include explicit learning experiences that promote this ideal.

This course uses an asset-based framing and the instructors understand that all students enter the course with some amount of mathematics knowledge. “To make learning stick, we have to determine what students already know and understand how they have organized it in their schema” (Hammond, 2015, p. 49). The course was designed so that students and teachers are aware of what the students know, what the students need to learn, and what misconceptions the students have. Then, building on the prior knowledge, the students can develop an understanding of the mathematics concepts. The instructors need to be aware that “whatever the neuronal networks are in the student brain, the teacher cannot remove them” (Zull, 2002, p. 101). But, teachers can “reduce the use of particular networks, or to use other networks in their place, and some networks may die out or weaken with disuse” (p. 101).

One way for students to gain a conceptual understanding is for students to develop an understanding of the connections among mathematical objects. As students “learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding” (National Council of Teachers of Mathematics, 2014, p. 24). Developing this deeper understanding begins with students’ ability to use different representations of mathematical concepts (Hammond, 2015; National Council of Teachers of Mathematics, 2014; Zull, 2004). In fact, Zull (2004, p. 146) suggests, “that teachers could make extensive use of images to help people learn.” He goes on to recommend “And whenever possible, we should require our students to show us their images” (p. 146). The motivation for students to use multiple representations in the learning process is that it involves different areas of the brain. As different areas are being used, stronger neural pathways develop and, with them, a deeper understanding.

The learning experiences should be designed with purposeful activities that promote building on prior knowledge and using multiple representations to develop a deeper understanding. These activities consist of both in-class experiences and out-of-class experiences, where both work together to enable students to see connections between mathematical objects. Weimer (2013) describes well-designed learning experiences as:

1. These learning experiences (be they assignments or activities) motivate student involvement and participation.
2. One of the best ways to accomplish the first objective is with assignments and activities that get students doing the authentic and legitimate work of the discipline.
3. Well-designed assignments and activities take students from their current knowledge and skill level to a new place of competence, and they do so without being too easy or difficult.
4. These are experiences that simultaneously develop current knowledge and learning skills. (p. 77)

It is not enough for the students to see the mathematics done by the instructor; instead, the students need to practice the activities themselves (Hammond, 2015; National Council of Teachers of Mathematics, 2014). As "effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (National Council of Teachers of Mathematics, 2014, p. 17). Furthermore, *Principles to Actions* suggests, "the role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others" (p. 11).

In practice, this developmental mathematics class uses a flipped-classroom design to initiate the learning. The students are expected to watch a video that emphasizes the connections between mathematical objects before each class session. A workbook is associated with the video, in which the students answer questions based on the material in the video, and purposeful problems that are used to identify weakness and misconceptions the students have. When students come to class, they discuss any problems that arose while they were watching the video that they did not understand. Then the students work on in-class activities that promote conceptual understanding of the material. Finally, after class is complete, the students complete an online homework assignment that focuses on the procedural fluency and gives immediate feedback after each problem worked.

During class time, the instructor facilitates open discourse, in which mistakes are celebrated and learned from. Students can ask questions and come to their own understanding. Each of the activities is designed with a learning outcome in mind and is accessible to all learners in the classroom. In addition, the activities challenge the students to form multiple representations of mathematical ideas.

The design of the mathematical content of this course starts with the learning outcomes that focus on both conceptual understanding and the use of that understanding to build procedural fluency. To promote connections among mathematical objects, the topics in the course are taught in a different order than in a typical developmental mathematics classroom. (See Appendix E for complete table of contents examples of both.) The first chapter of the content focuses on definitions of the mathematical objects used throughout the course. Once the definitions are understood, the subsequent material uses these definitions to develop the understanding between mathematical objects.

To understand how this design principle is realized in the classroom, I used a pre-test of conceptual understanding and procedural fluency. The test took place in the first week of class before formal instruction began. This test focused on integers, fractions, and operations with these objects because many of the students in this course have a fragmented understanding of these concepts and the procedures related to them but struggle with the conceptual understanding (Stigler et al., 2010). To assess if there is a change in understanding, questions from the pre-tests appeared on the in-class final. By having the questions embedded in the final exam, students were less aware that these are research questions, and the assessment did not take additional class time. Since these students have all had exposure to the material in the class, a change in understanding can be attributed to the course. In addition to the testing, samples from the workbook and classroom activities are included (see Appendix G and Appendix H). Finally, videotaped classroom episodes were analyzed for examples of mathematical discourse.

4.3 Design Principle 3

Provide a variety of learning opportunities about different affective characteristics to promote a positive attitude change in students' mathematical mindset.

In addition to fragmented understanding, many students entering a developmental classroom also have difficulties learning how to learn mathematics. Jo Boaler (2015) says,

> "We need to free our young people from the crippling idea that they must not fail, that they cannot mess up, that only some students can be good at math, and that success should be easy and not involve effort. We need to introduce them to creative, beautiful mathematics that allows them to ask questions that have not been asked, and to think of ideas that go beyond traditional and imaginary boundaries." (p. 208)

Learning mathematics can be messy and difficult; as instructors, we need to give students the tools to enable them to succeed in places they may have failed before. To this end, in addition to mathematical content, students also need to learn how to learn mathematics (Hammond, 2015; Weimer, 2013).

Students need to develop their own beliefs that they can succeed in their mathematical learning. This can be accomplished through growth mindset instruction (Boaler, 2015; Dweck, 2008; National Council of Teachers of Mathematics, 2014), and facilitating a belief they are capable of learning mathematics (Hammond, 2015; National Council of Teachers of Mathematics, 2014).

Finally, students need to take responsibility for their own learning. Since this may be a new concept for many students, they will need help at first (Hammond, 2015). When students take responsibility for their own learning, they are able to identify their own strengths and weaknesses. This provides a framework for discussion in the class.

In practice, this is done by specific instruction about notetaking, studying, and test-taking. Nolting (2014) explains that learning math is different than learning other subjects. As such, students should be taught how to take notes in a math class. Therefore, we spend time in class and have assignments that show students how to take effective notes for math class. This includes not only writing what the instructor puts on the board (or videos) but also what the instructor and students discuss in class.

Students are given the learning outcomes for each class session and asked to self-assess where they are in their learning of these learning outcomes. As students become aware of what they need to work on and what they understand, class time is tailored to fit the students. If the entire class is struggling on a concept, more class time is devoted to the material. This helps all students gain a better understanding of the material. If only a couple of the students are struggling, the instructor can give personalized instruction to those students during class activity time or office hours.

Students are encouraged to do an error analysis daily during our classroom discussions and again for all exams taken in class. Weimer (2013, p. 187) states, “students regularly report that they learn much more when they have to correct their own errors than when they listen to the teacher explain the right answer.” Therefore, if student learning is at the forefront of class design, students should be held accountable for correcting their own work.

Finally, during the course of the semester, students are expected to watch different videos highlighting a variety of affective characteristics. These videos include people talking about grit, growth mindset, time management, and note-taking. As Weimer (2013) states, “Sometimes the messages about how to learn are more effective when they come from someone other than the professor” (pp. 134-135).

To understand how this design principle is realized in the classroom, I had the students complete a math attitude survey at the beginning and end of the course. Also, during the focus group there were questions on students’ beliefs about the effectiveness of the instruction on notetaking, studying and test-taking, along with their beliefs about the effectiveness of the videos and follow-up conversations.

4.4 Design Principle 4

Create an assessment system that explicitly values learning including cognitive and social-emotional outcomes.

Since this is a class, there needs to be a course grade at the end of the semester. “Doing the work required in a course gets students grades, but it’s also an opportunity for them to learn” (Weimer, 2013, p. 176). This grade should value a student’s progress toward the learning outcomes of the course, not an evaluation of what they can remember on one day at the end of the course. Weimer (2013) has four principles for the relationship between grades and learning. First, harness the power of grades to motivate the students. Second, make evaluation experiences less stressful. Third, use evaluation only to assess learning. Fourth, focus more on formative feedback. Using these principles, it is possible to create an assessment system that values learning.

The most important aspect of assessment is quality, on-time feedback. Students need feedback or they will continue to do the same work over and over (Hammond, 2015). Hammond goes on to describe quality feedback as instructive rather than evaluative, specific and in the right dose, timely, and delivered in a low-stress, supportive environment. Therefore, an emphasis in the course should be placed on formative feedback where students have the ability to learn from the feedback. Boaler (2015) suggests that students should always be allowed to resubmit work for a higher grade.

Formative assessments are necessary if creating an assessment system that values learning. "Waiting until the quiz on Friday or the unit test to find out whether students are making adequate progress is too late" (National Council of Teachers of Mathematics, 2014). Instead, regular timely feedback helps students learn from their mistakes and move forward with their learning.

In practice, the assessment system for this course is multi-dimensional. This course is not graded as three tests and a final; rather the final grade comprises all aspects of the course including participation and formative and summative assessments because each part is integral in a student's learning. Each of the items that carry a weight for the course is designed for student learning compared to grading only a student's summative assessments. In addition, students have course requirements that value their learning. Each of the formative assessment items is given a "meets expectations" evaluation or a "not yet" evaluation with feedback about what should be done to make corrections. Students are given the opportunity to correct every formative assessment until it "meets expectations." Each of the summative assessment items is evaluated on a four-point scale – "exceeds expectations," "meets expectations," "developing," or "needs improvement" – based on a specific learning outcome for the course. Students are given the opportunity to retake each of the summative assessments once (not including the final exam) if the overall score is lower than "meets expectations."

The first way to see this in practice is that coming prepared to class and participating in class make up a large portion of the final grade. This extrinsic motivation to come to class leads to an intrinsic motivation when students understand the learning benefits that occur in class.

There are assignments in the course that promote the affective learning that takes place in the course. These all have a small weight toward a student's final grade. This gives evidence to the students that these exercises are as important as learning the mathematics. These assignments include exam corrections and conferencing with their instructor.

As described in Design Principle 2, students have workbook assignments and in-class assignments that are used to promote conceptual understanding. As a preparation for summative assessments, students are required to have all workbook assignments and in-class assignments turned in, complete and correct, to be eligible to take the summative assessment. Students are given the opportunity to ask questions in class or during office hours about the assignments and are allowed to make corrections to the assignments until they are correct.

When explaining this policy to students, instructors ask students why they should take an exam they are not prepared to take – instead, if they do their assignments, they will be prepared to take the exam.

Students have three mid-term summative assessments. Each of these assessments is untimed. In addition, students are allowed to retake the assessment if they scored below an 80%. This lets students know that the priority of the course is their learning, not their performance on one or two exams. This also helps to prevent students who have a bad test from giving up (Ehlert, 2015). The students are able to spend some time on the material they were struggling with and reassess a week later and improve their score.

To understand how this is realized in the course, the syllabus for the course is included (see Appendix J). This will provide evidence of the course grading policy. In addition, example rubrics of both formative and summative assessment focusing on the fraction and integer content are included (see Appendix I).

4.5 Conclusion

Students who are referred to multiple developmental mathematics classes have a severe history of learning mathematics. This course is designed using research-based principles to empower these students to succeed at mathematics. All of the design principles work together to build a holistic class. This holistic class allows students who have had a troubled history in mathematics to gain a better understanding of basic mathematics and what it takes to gain this understanding. At the conclusion of this class, the students can use the habits and understandings from this class to progress on their collegiate path. Table 4 is a summary of the design principles and how each principle is measured.

Table 4

Summary of the Design Principles and How They are Measured

Design Principle	Design Principle 1: Equitable Environment	Design Principle 2: Conceptual Understanding Building Procedural Fluency	Design Principle 3: Affective Characteristic Instruction	Design Principle 4: Assessment System
Research Questions	Q3a: Change in discourse throughout the semester	*Q1:* Effect on student understanding *Q3d:* Student perception of class activities to promote conceptual understanding	*Q2:* Effect on students' attitude toward mathematics *Q3c:* Student perception of growth mindset activities	*Q3a:* Student perception of most important feature *Q3b:* Student perception of learner-centered assessment system
Surveys and Tests	Student Experience Survey	Pre-test of integer, fractions and their operations Final exam questions associated with the pre-test	ATMI – At the beginning and end of the semester	Student Experience Survey
Classroom Artifacts	Initial email exchange, phone script, and notes, shared class expectations	Sample workbook sections and select classroom activities		Course syllabus with assessment system, example rubrics, student artifacts
Videotaped Classroom Observations	Excerpts of evidence of	Excerpts of evidence of		

Design Principle	Design Principle 1: Equitable Environment	Design Principle 2: Conceptual Understanding Building Procedural Fluency	Design Principle 3: Affective Characteristic Instruction	Design Principle 4: Assessment System
	classroom connections	mathematical discourse		
Interviews and Focus Group	Questions on classroom connectedness		Questions on affective characteristics instruction	Questions on value of assessment system

Chapter 5 Results

The beginning of the results section will contain a brief description of the design principles of the course.

1. Create an equitable environment where students are comfortable participating in mathematical discourse.
2. Organize learning outcomes and class activities in a way that promotes conceptual understanding and procedural fluency among mathematical objects.
3. Provide a variety of learning opportunities about different affective characteristics to promote a positive attitude change in students' mathematical mindset.
4. Create an assessment that explicitly values learning including cognitive and social-emotional outcomes.

Then the results section will address how each research question is measuring different aspects of each design principle.

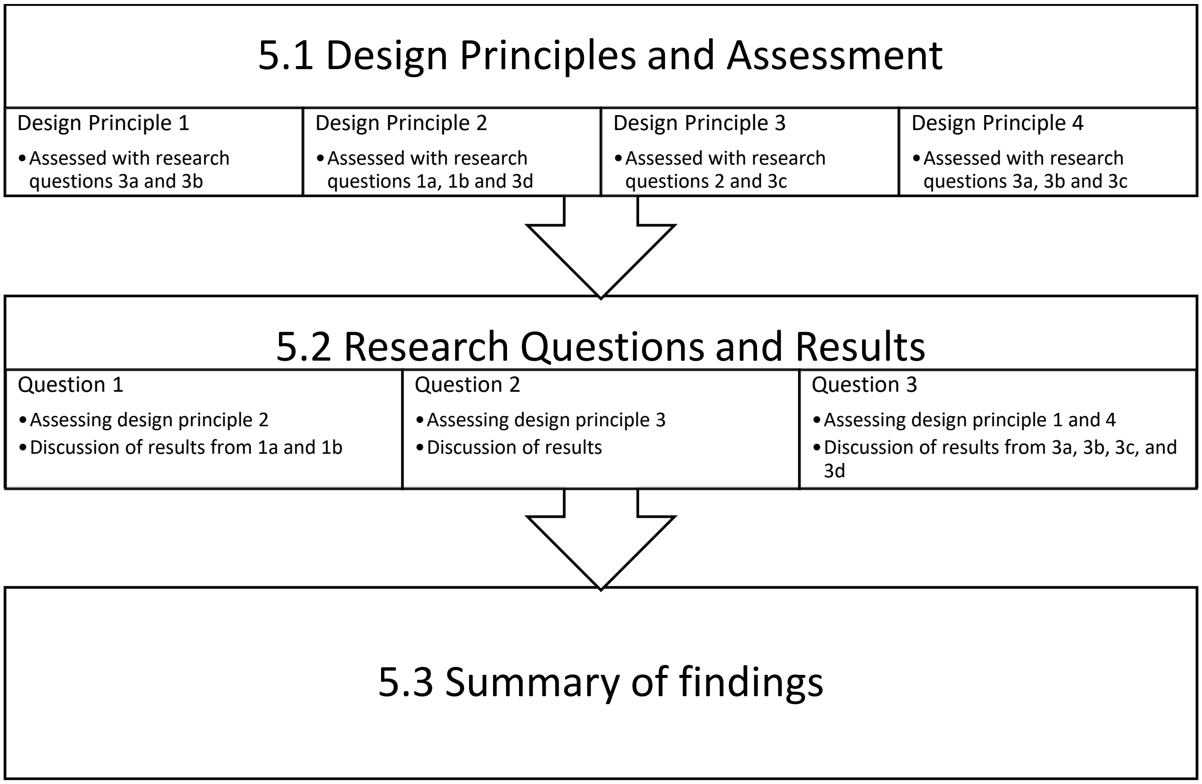
1. How do the design changes affect student learning? In particular:
    a. How does targeted conceptual understanding instruction affect a student's conceptual understanding of and procedural fluency in integers and fractions and their operations? (This question is measuring Design Principle 2)
    b. How do the design changes of the course compare success rates of current student to historic data from students before the reform took place? (This question is measuring Design Principle 2)

2. How do the design changes affect students' attitude toward mathematics, and is there a change in discourse habits over the course of the semester? (This question is measuring Design Principle 3)
3. What are the students' perceptions of the course design? In particular:
    a. What features of the course design do students believe is most important for their learning? (This question is measuring Design Principle 1 and 4)
    b. What are the students' perceptions of learner-centered assessment system? (This question is measuring Design Principle 4)
    c. What are students' perceptions of the learning opportunities designed to promote a growth mindset? (This question is measuring Design Principle 4)
    d. What are students' perceptions of learning opportunities to promote a conceptual understanding of integers and fractions and their operations? (This question is measuring Design Principle 2)

Finally, I will discuss the findings in relation to the research questions. Figure 5 is a roadmap of how chapter five is organized.

Figure 5

Results Section Roadmap



5.1 Design principles of the course

The course was designed to provide support for entry into credit-bearing university mathematics for students with a severe history in their mathematics education. These students are likely to: have a fragmented knowledge of mathematics (“ACT College and Career Readiness Standards,” 2017); have a fragile procedural knowledge of basic mathematics (Stigler et al., 2009, 2010); be first-generation, first-year students; and/or have outside factors that interfere with the success in mathematics courses (Acee et al., 2017). The course is designed to empower students to remove barriers to their success and provide them with a path for success in mathematics.

The following sections briefly discuss the four design principles for the course. The design principles are research-based practices put together to address developmental mathematics students' history in mathematics. In addition to the discussion of the principles, I will discuss how the design principles are realized in the course. This is including brief examples of what this looks like in the course. Last, for each design principle I will discuss how it will be measured in the research questions.

5.1.1 Design Principle 1: Create an equitable environment where students are comfortable participating in mathematical discourse

The most important part of the course design is designing an environment where students are comfortable learning. The learning opportunities of the course are designed to be accessible for all learners and provide challenge for all learners. The design of the course provides opportunities for the instructors to develop connections with the students and the students to develop connections with other students in their class. In addition, the course offers opportunities for students to make connections with each other.

The course begins with a two-day instructor training the week before the semester begins. The first day focuses on the instructor experiencing what the student experiences the first day of class. Instructors go through the first day activities and talk about the different responses that they will see from students. The instructors also take part in some of the conceptual learning activities that will take place throughout the semester. For many of the instructors that are new to the course these activities are new and provide an opportunity for the instructors to struggle similar to how the students will struggle. Thus, they will understand some of the challenges students face when presented with different ways to approach mathematical thinking.

The semester begins with an email (see Appendix F) from the instructor introducing themselves and the course. Students are asked to complete a student survey to start the course, and instructors are encouraged to follow-up with the students from the result of the survey. The first day of class is spent introducing students to the course, and more importantly introducing students to each other and facilitating a collaborative environment from day one. Throughout the semester, the instructor facilitates discussions daily. The discussions include student's daily work, various in-class activities, and individual discussions with the instructor.

Research Question 2 in part measures the implementation of this design principle: *How do the design changes affect students' attitude toward mathematics, and is there a change in discourse habits over the course of the semester?* I have included the templates of the emails to the student, the initial survey and the phone script in Appendix F. To determine the change in the discourse habits, I looked at classroom observations for student interactions with each other and student interactions with the teacher. During the observations I was looking for students to be engaged during group work, talking to each other, and explaining things to each other. During the observations, I recorded how often students were engaged in the class activities, what types of questions the instructor was asking, and the types of encouragement teachers were giving students. In addition, I was looking for instructors to facilitate student discussion promote a sense of belonging in the class. Last, I conducted two focus groups at the end of the semester and the beginning of the next semester. When analyzing the focus groups, I looked for evidence of the students talking about their relationship with their teacher, and their relationship with other students.

5.1.2 Design Principle 2: Organize learning outcomes and class activities in a way that promotes conceptual understanding and procedural fluency among mathematical objects

Every student that is referred to this course has knowledge of the mathematics in this course. Included in this knowledge is various misconceptions and fragmented understanding (Stigler et al., 2009, 2010). Stigler et. al. (2009) found students had a reliance on procedures they did not fully understand. This course is designed to incorporate the knowledge students come to the course with, address their misconceptions through discovery (Zull, 2002), and increase their conceptual understanding.

The learning experiences in the class are multi-faceted. Students watch videos before class to initiate their learning process. The videos talk about the definitions and basics of each topic. Each of the videos ranges from 15 minutes to 45 minutes on average. Each video attempts to draw connections between the topics in the section, compared to the quick five-minute videos that address a single topic. The students are required to take notes on the videos in the associated workbook section. After watching the video, the students are required to attempt the workbook exercises. At the beginning of class each student's workbook section is checked for completion and they earn grade points if the work is completed. During class students are asked to put their work on the board either individually, as pairs, or as groups depending on the exercises. If students have not completed some of the exercises, they are paired up with students that did complete the exercise so they can discuss as they put the work on the board. After the work is on the board, the instructor facilitates a discussion about the work. If there are mistakes in the work, the class discusses what the mistake is, and why it was made. This open discussion promotes a conceptual understanding as students are able to learn from their own or other students' mistakes. The instructor is able to discuss the underlying misconception and students are able to take notes. As the class is moving through each workbook section, students are encouraged to correct mistakes and make notes on why they had the mistake if they want to. Once the class has completed a discussion on the workbook exercises, students are put into groups to discuss the application and discussion section. Each workbook section contains an application section that makes connections between individual learning outcomes, provides practical applications, or extends individual learning outcomes. The application section is designed to be completed together so students can discuss the mathematics. In addition to the application section, there is a discussion section that is worked on in groups. The discussion section has questions that address the overview of the section, common student misconceptions, and connections between different sections. Last, there is a review section in each workbook section for students to review concepts from earlier in the semester. The review section is made up of learning outcomes that students struggle with throughout the semester and previous learning outcomes that relate to the workbook section. (See Appendix G for an example of a workbook section.)

In addition to the design of the class day and workbook structure, the learning outcomes in the course are ordered to promote a conceptual understanding. To promote this conceptual understanding and incorporate the prior knowledge that students enter the course with, the topics in the course are taught in a different order than a typical developmental mathematics class. (See Appendix E for a complete table of contents from this course.)

Last, throughout the course, students are given activities that enhance their conceptual understanding of the learning outcomes of the course. I have included two activities that address the fraction and integer operations that were done in class the semester under study. (See Appendix H for the activities.)

This design principle was assessed with two different research questions. First, Research Question 1:

How do the design changes affect student learning? In particular:

a. How does targeted conceptual understanding instruction affect a student's conceptual understanding of and procedural fluency in integers and fractions and their operations?

To determine if there was a change in conceptual understanding and procedural fluency in integers and fractions and their operations students were given a pre-test of integers, fractions and their operations before any instruction on these topics were started. Then at the end of the semester, students were given a final exam with the same questions as the pre-test as part of the final exam. The results are analyzed using a repeated design t-statistic to determine if there is a significant difference in the pre-test and post-test. The scoring rubric for the individual learning outcomes is included in Appendix I.

b. How do the design changes of the course compare success rates of current students to historic data from students before the reform took place?

To determine if there is a change in success rates of students of the current students compared to students that took the sequence before the reform took place, I analyzed the pass rates of the current students in the course. I used ANOVA to determine if there was a difference in the pass rates of students with ACT Math level less than 16 compared to any of the other ACT Math levels. In addition, I did a deeper analysis of the students that earned a D, F or W in the class to determine how they did in their other classes. This was to determine if Math was the only class they did not pass.

Next, I looked at the pass rates of the students that continued in a math class the subsequent semester. I looked at the pass rates from the students that took a credit bearing math class and compared it to all the students that took the same class that semester. In addition, I compared the pass rates of the credit-bearing math class of the students in the study to the historic pass rates from pre-reform. The last semester that this class was taught in a traditional format was Fall 2012. Therefore, I asked for the pass rates of students that took the same class in Fall 2012 and the same students that were enrolled in a credit-bearing math class the following semester. The differences were tested using a t-statistic.

The other research question that assesses Design Principle 2 is Research Question 3d:

> What are students' perceptions of learning opportunities to promote a conceptual understanding of integers and fractions and their operations?

To determine the perception of the learning opportunities designed to promote a conceptual understanding of integers and fractions and their operations I analyzed the student survey responses and the focus groups. In the survey responses, I read the answers from the response questions and identified the responses that included information about the in-class activities, understanding of the material, or what the students perceived was important to their learning. Students were very direct in their responses and said that the activities were either beneficial or not beneficial. I coded the data based on their interpretation of their responses. For the responses that were more robust, I coded based on the meaning of their response.

5.1.3 Design Principle 3: Provide a variety of learning opportunities about different affective characteristics to promote a positive attitude change in students' mathematical mindset

The design of this course includes providing instruction on promoting a growth mindset, productive struggle, time-management, note-taking and test-taking instruction. Some of this instruction is transparent and visible to students. This includes watching videos in class and in discussion that discuss these affective characteristics. Included TED talks such as:

- Sal Khan – Let's teach for mastery
- Angel Duckworth – Grit
- Laura Vanderkam – How to gain control of your free time
- Kelly McGonigal – How to make stress your friend
- Roger Antonsen – Change your perspective

Additionally, students watched videos in their discussion section about math study skills. The videos were created by Bernards and DeSmet and used with permission from the authors. After the watching the videos students would either have a discussion about the video or complete a short assignment detailing what they learned in the videos.

Conversely, there are many aspects of the design of the course that were not visible instruction on affective characteristics. The most important of these is the instructor training that occurred at the beginning of the semester to help instructors realize that all students in the course have the ability to succeed in this class. In addition, there is an emphasis placed on students learning from their mistakes and the mistakes of others in the class. Students are required to revise their daily work until it is correct. This usually occurs with one revision considering the work is discussed during class every day. Students are also required to complete exam corrections on all exams except the final. This is used as a learning experience, so students fully understand the material. Furthermore, attendance is mandatory for class and weekly discussion. Last, the workbook includes a place for students to complete notes on every video, and a place for students to self-asses their strengths of the section (See Appendix G for an example of the workbook section.)

This design principle will be assessed with two different research questions. The first question is Research Question 2:

> How do the design changes affect students' attitude toward mathematics, and is there a change in the discourse habits over the course of the semester?

To determine if there is a change in students attitudes toward mathematics, students were given the Attitude Toward Mathematics Index (Tapia, 1996). I received permission from the author to use the index for research purposes. The students answered the index questions at the beginning of the semester and again at the end of the semester. I performed a t-test to determine if there was a change in students' attitude toward mathematics at the end of the semester compared to the beginning of the semester.

This design principle will also be assessed in Research Question 3c:

> What are students' perceptions of the learning opportunities designed to promote a growth mindset?

To determine the students' perceptions of these learning opportunities, I analyzed students' responses to the Student Impression Survey at the end of the semester (See Appendix B). These results were analyzed both quantitively using descriptive statistics of students ranking and the Likert questions and qualitatively using students' responses to the four short answer questions. This analysis is supplemented by students' responses in the focus group. In the analysis, I was focusing on what students ranked as most important and least important for their learning. Also, I looked at the student responses for mentions of why they felt the different aspects were important or not important to their learning. The items that address promoting a growth mindset using a Likert scale were:

- The worksheet corrections helped strengthen my understanding of mathematics
- Knowing that I have to qualify to take exams better prepares me for the exams
- I feel there is an emphasis placed on learning in this course
- Self-correcting my mistakes helps me understand my misconceptions
- The study skill videos I watched in discussion helped me with my study skills
- I am better able to self-assess if I have fully understood a topic or if I need more instruction on the topic
- I feel there is an emphasis placed on learning from mistakes in this course
- The TED talk videos we watched in class helped me understand how to learn math

These items were analyzed using descriptive statistics to determine students' perceptions of their importance.

Last, the student short answer responses and focus group responses were analyzed to determine students' perceptions of the aspects of the course that promote a growth mindset. The items were coded if they included mention of any aspect of the course that promoted a growth mindset or affective characteristics. Then, the data was analyzed to determine if any themes emerged.

5.1.4 Design Principle 4: Create an assessment system that explicitly values learning including cognitive and social-emotional outcomes

The assessment system in this course places an emphasis on the activities that will promote student learning. Therefore, all aspects of the course have grade points assigned to them if they promote student learning (Weimer, 2013). This includes the importance placed on attendance, coming to class prepared, and formative assessments of workbook completion. As students perform the formative items, they have the understanding they need to perform well on the summative assessment. (See Appendix J for the syllabus for the course)

Student perception of the grading system is important in the course design. If students see the grading system as beneficial to their learning, they will direct their efforts towards the elements of the course valued by the grading system in part because of the gatekeeping function of the course. This design principle will be assessed by two research questions. The first question is Research Question 3a:

What features of the course design do students believe are most important for their learning?

This is analyzed by combining the data students used to rank the importance of course aspects with their responses to the short answer questions. By combining these two measures we can gain a deeper understanding of why students feel some aspects of the course are more important than others. Furthermore, the focus group responses were analyzed and when aspects of the grading system were mentioned the instances were coded and analyzed with the responses from the student survey results.

This design principle is also assessed in Research Question 3b:

What are the students' perceptions of the learner-centered assessment system?

This question is analyzed using the Student Impression Survey in a variety of ways. First, there are multiple Likert questions that address the importance of the assessment system including:

- Knowing that I have to qualify to take exams better prepares me for the exams
- I feel there is an emphasis placed on learning in this course
- Self-correcting my mistakes helps me understand my misconceptions
- I feel the grading system values my learning
- I feel that all work I complete in the course has a value
- I am glad I have the ability to re-do an exam if I do not perform well the first time

These were analyzed using descriptive statistics to determine their relative importance.

Additionally, the students' ranked different aspects of the course in regard to their importance. The results were analyzed by combining the student ranking and their short answer responses to gain a better understanding of why students feel some aspects are more important than other aspects of the course.

Now I will go on to discuss the research questions and the findings from each of the research questions.

5.2 Research Questions and Results

1. How do the design changes affect student learning? In particular:

a. How does targeted conceptual understanding instruction affect a student's conceptual understanding of and procedural fluency in integers and fractions and their operations? (This question is measuring Design Principle 2)

b. How do the design changes of the course compare success rates of current student to historic data from students before the reform took place? (This question is measuring Design Principle 2)

2. How do the design changes affect students' attitude toward mathematics, and is there a change in discourse habits over the course of the semester? (This question is measuring Design Principle 3)

3. What are the students' perceptions of the course design? In particular:

    a. What features of the course design do students believe is most important for their learning? (This question is measuring Design Principle 1 and 4)

    b. What are the students' perceptions of learner-centered assessment system? (This question is measuring Design Principle 4)

    c. What are students' perceptions of the learning opportunities designed to promote a growth mindset? (This question is measuring Design Principle 4)

    d. What are students' perceptions of learning opportunities to promote a conceptual understanding of integers and fractions and their operations? (This question is measuring Design Principle 2)

5.2.1 Research Question 1: How do the design changes affect student learning?

5.2.1.1 How does targeted conceptual understanding instruction affect a student's conceptual understanding of and procedural fluency in integers and fractions and their

operations. I created a pre-test and questions for the final exam that focused on fractions, integers and their operations. The test was looked at by two colleagues to determine content validity. The final exam contained the exact questions from the pre-test and some other questions from later in the semester. The pre-test was given at the end of the first week of class, before any instruction on fractions or integers began. The final exam was given during finals week at the end of the semester. Students were not allowed to use a calculator for either of the exams. (See Appendix C for the pre-test and Appendix I for the grading rubric.) In general, a score of 4 on a learning outcome shows the student is very proficient in the learning outcome, a score of 3 shows that the student is proficient in the learning outcome, a score of 2 shows some understanding but the student has major misconceptions, a score of 1 indicates little understanding and a score of 0 indicates the student did not attempt the problem.

Initially I looked at the proficiency levels for each of the individual learning outcomes (see Table 5). A mean score of 3 or more indicates that the students are proficient in the learning outcome. A score of less than 1.5 indicates the students have little understanding of the learning outcomes. On the pre-test students showed proficiency on only two of the learning outcomes, plotting integers on a number line and multiplying and dividing integers (see Table 5 or Figure 6). The pre-test also indicated that students had little understanding of six of the learning outcomes, including modeling fractions, plotting fractions on a number line, determining the distance between two fractions, multiplying rational expressions, dividing fractions, and dividing rational expressions. In the analysis fractions are rational numbers that have integer numerators and denominators, and rational expressions include variables. The analysis of the final exam learning outcomes show that students show a basic understanding on nine learning outcomes: *modeling rational expressions, plotting integers on a number line, finding the distance between two integers on a number line, determining if two fractions are equivalent, ordering fractions, adding and subtracting fractions, multiplying fractions, adding and subtracting integers*, and *multiplying and dividing integers*. Additionally, there were two learning outcomes on the final that had a mean score of over 2.9. These were *multiplying rational expressions* and *dividing fractions*. The learning outcomes of *plotting fractions on a number line*, *finding the distance between two fractions on a number line*, *determining the decimal equivalent of a fraction*, and *dividing rational expressions* showed that students had a limited understanding. The learning outcome *determine the decimal equivalent* may be skewed because the denominators of the fractions they needed to convert were not all commonly used denominators, for instance they had to convert sixths and eighths to decimal equivalents without a calculator. Figure 6 shows the pre-test and final exam scores as a bar chart with the standard error graphed. The gains for each of the learning outcomes can be seen in Figure 6.

Table 5

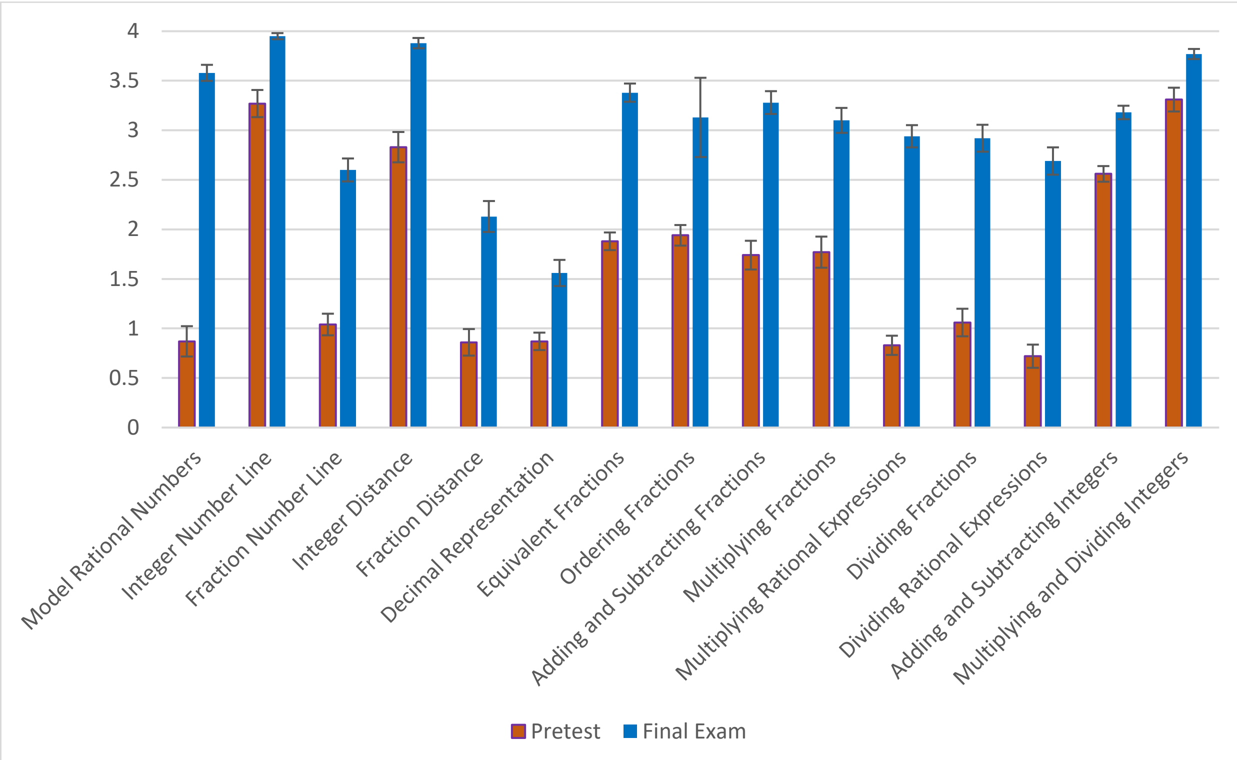
Learning Outcomes Pre-Test and Final Exam Descriptive Statistics

	Mean	N	Std. Deviation	Std. Error Mean
Final exam total	46.10	78	8.49	.96
Pre-test total	25.56	78	6.95	.79
Final model rational numbers	3.58	78	.71	.08
Pre-test model rational numbers	.87	78	1.35	.15
Final integer number line	3.95	78	.27	.03
Pre-test integer number line	3.27	78	1.21	.14
Final fraction number line	2.60	78	1.02	.12
Pre-test fraction number line	1.04	78	.96	.11
Final integer distance	3.88	78	.46	.05
Pre-test integer distance	2.83	78	1.35	.15

	Mean	N	Std. Deviation	Std. Error Mean
Final fractional distance	2.13	78	1.38	.16
Pre-test fractional distance	.86	78	1.18	.13
Final decimal representation	1.56	78	1.17	.13
Pre-test decimal representation	.87	78	.78	.09
Final equivalent fractions	3.38	78	.81	.09
Pre-test equivalent fractions	1.88	78	.79	.09
Final ordering fractions	3.13	78	3.52	.40
Pre-test ordering fractions	1.94	78	.91	.10
Final adding and subtracting fractions	3.28	78	1.01	.12
Pre-test adding and subtracting fractions	1.74	78	1.28	.15
Final multiplying and dividing fractions	3.10	78	1.11	.13
Pre-test multiplying and dividing fractions	1.77	78	1.39	.16
Final multiplying rational expressions	2.94	78	.99	.11
Pre-test multiplying rational expressions	.83	78	.86	.10
Final dividing fractions	2.92	78	1.20	.14
Pre-test dividing fractions	1.06	78	1.23	.14
Final dividing rational expressions	2.69	78	1.22	.14
Pre-test dividing rational expressions	.72	78	1.03	.12
Final adding and subtracting integers	3.18	78	.60	.07
Pre-test adding and subtracting integers	2.56	78	.70	.09
Final multiplying and dividing integers	3.77	78	.45	.05
Pre-test multiplying and dividing integers	3.31	78	1.06	.12

Figure 6

Change in Learning Outcomes with Standard Error



After looking at the proficiency levels, I ran a paired t-test comparing the cumulative score on the pre-test and the cumulative score of the same questions on the final exam. Additionally, I ran a paired t-test for each of the individual learning outcomes. All comparisons showed a statistically significant difference between the cumulative score on the pre-test and the cumulative score on the final (see Table 6). Additionally, because of the number of tests, I ran a false rate discovery test to adjust the alpha levels and control for Type I error (see Table 8). All the paired t-tests remained significant. The effect size of the cumulative score test is $r^2 = 0.84$ indicating 84% of the variance between the two tests can be accounted for by the course. *Ordering fractions* and *multiplying and dividing fractions* had a small effect when

Cohen's d is calculated (see Table 7). *Plotting integers on a number line* and c*onverting fractions to decimal number* had a medium effect. All other learning outcomes showed a large effect when Cohen's d is calculated between the final and the pre-test. These results indicate the course does have a significant impact on students' conceptual understanding. Figure 7 shows a chart of the mean difference in order of lowest to highest. Note, the learning outcomes with the smallest difference scores were the learning outcomes with the greatest initial proficiency. Next I will analyze pass rates for the course.

Table 6

Pre-test and final exam comparison

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	Total Score	20.54	8.91	1.001	18.53	22.55	20.35	77	.000***
Pair 2	Model rational numbers	2.71	1.42	.16	2.39	3.02	16.88	77	.000***
Pair 3	Integer number line	.68	1.21	.14	.41	.95	4.95	77	.000***
Pair 4	Fraction number line	1.56	1.17	.13	1.30	1.83	11.82	77	.000***
Pair 5	Integer distance	1.05	1.30	.15	.76	1.34	7.15	77	.000***
Pair 6	Fractional distance	1.27	1.40	.16	.95	1.59	7.00	77	.000***
Pair 7	Decimal representation	.69	1.26	.14	.41	.98	4.85	77	.000***
Pair 8	Equivalent fractions	1.50	1.00	.11	1.27	1.73	13.21	77	.000***
Pair 9	Ordering fractions	1.19	3.49	.40	.40	1.98	3.01	77	.003**
Pair 10	Adding and subtracting fractions	1.54	1.34	.15	1.24	1.84	10.17	77	.000***
Pair 11	Multiplying and dividing fractions	1.33	1.63	.18	.97	1.70	7.25	77	.000***
Pair 12	Multiplying rational expressions	2.10	1.21	.14	1.83	2.38	15.32	77	.000***
Pair 13	Dividing fractions	1.86	1.54	.17	1.51	2.21	10.70	77	.000***

Pair 14	Dividing rational expressions	1.97	1.60	.18	1.62	2.33	10.93	77	.000***
Pair 15	Adding and subtracting integers	.62	.74	.08	.45	.78	7.32	77	.000***
Pair 16	Multiplying and dividing integers	.46	1.13	.13	.21	.72	3.63	77	.001**

**p<.01, ***p<.0001

Table 7

Effect size of the individual learning outcomes

Learning Outcome	Cohen's d
Modeling rational numbers	1.91
Plotting integers on a number line	0.56
Plotting fractions on number line	1.33
Determining the distance between two Integers	0.80
Determine the distance between two fractions	0.90
Converting a fraction into a decimal number	0.54
Identifying equivalent fractions	1.49
Ordering fractions	0.34
Adding and subtracting fractions	1.15
Multiplying fractions	0.82
Multiplying rational expressions	1.73
Dividing fractions	1.21
Dividing rational expressions	1.23
Adding and subtracting integers	0.86
Multiplying and dividing integers	0.41

Figure 7

Mean Difference in Pre-Test and Final Exam Scores

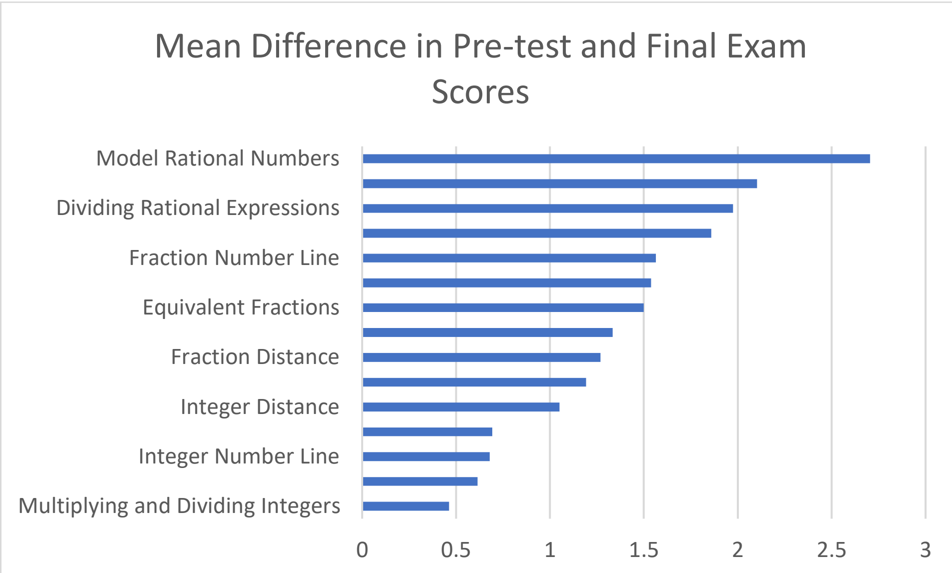


Table 8

Adjusted alpha-levels using false discovery rate adjustment

Original p-value	Index	q	m	Adjusted p-value	Significant
0.00	16	0.05	16	0.05	1
0.00	15	0.05	16	0.047	1
0.00	14	0.05	16	0.044	1
0.00	13	0.05	16	0.041	1
0.00	12	0.05	16	0.038	1
0.00	11	0.05	16	0.034	1
0.00	10	0.05	16	0.031	1
0.00	9	0.05	16	0.028	1
0.00	8	0.05	16	0.025	1
0.00	7	0.05	16	0.022	1
0.00	6	0.05	16	0.019	1
0.00	5	0.05	16	0.016	1
0.00	4	0.05	16	0.013	1
0.00	3	0.05	16	0.0094	1
0.00	2	0.05	16	0.0063	1
0.00	1	0.05	16	0.0031	1

5.2.1.2 How do the design changes of the course compare success rates of current students to historic data from students before the reform took place. Starting the analysis for success rates, I did some initial coding. I coded the pass rates as 1 for an A, B, C grade because the student passed and is able to move to a credit bearing course. Then I coded a 0 as a D, F, or W grade because the student did not pass and is not able to move a credit bearing course. (A D is not a passing grade because the student does not meet the prerequisites for the credit-bearing course.) The overall pass rate for the students in the study was 79.5%. Students in this course are referred into the course because they earned a 0 on the math portion of the Placement Exam, or they had a 16 or less on their ACT Math score. Therefore, there are students in the class that have an ACT Math score of over 16. Thus, I disaggregated the data to determine if the students with different ACT Math score pass at different rates. I coded the students with an ACT of 16 or less as 16, students that earned a 17, 18, or 19 were coded as their ACT Math score, and students with an ACT Math score of 20 or more were coded as 20 (See Table 9 for the descriptive statistics of each group). Recall each student was scored as a 1 for pass and a 0 for did not pass. Thus, the students that started the course with ACT Math score of 16 had an 81% pass rate. According to the results from the one-way ANOVA there was no difference of pass rate for students based on their ACT Math scores, $F(4,74)=.261$, $p=.902$ (see Table 10). In addition, I ran a Levene's Test for homogeneity of variance of the group to check the equal variance assumption for ANOVA. The test showed that the variances of the ACT Math groups were equal (see Table 11). These results are different than the results from Tennessee. In the co-requisite model that Tennessee implemented, students with an ACT Math score of 16 or less has a slight increase in pass rate but, the pass rate was still low ("TBR CoRequisite Study - Update Spring 2016," 2016). These results are similar to the results shown in the Georgia study, but all three pathways were combined in the Georgia study (Denley, 2017). It is unclear how students performed in the algebra pathway only.

Table 9

Descriptive statistics for ACT Math levels (study group)

Pass

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
<=16.00	37	.81	.40	.07	.68	.94	.00	1.00
17.00	22	.82	.39	.08	.64	.99	.00	1.00
18.00	9	.67	.50	.17	.28	1.05	.00	1.00
19.00	6	.83	.41	.17	.40	1.26	.00	1.00
>=20.00	5	.80	.45	.20	.24	1.36	.00	1.00
Total	79	.80	.40	.05	.71	.89	.00	1.00

Table 10

ANOVA source table for ACT Math groups (study group)

ANOVA

Pass

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.178	4	.044	.261	.902
Within Groups	12.582	74	.170		
Total	12.759	78			

Table 11

Levene's test of homogeneity of variances of pass rates based on ACT Math scores (study group)

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Pass Rate	Based on Mean	.70	4	74	.59
	Based on Median	.26	4	74	.90
	Based on Median and with adjusted df	.26	4	71.8	.90
	Based on trimmed mean	.70	4	74	.59

In addition to overall pass rates of all students, of particular importance is disaggregated data based on student's race. To determine if there was a difference in pass rates, I performed a one-way ANOVA on pass rate using race as a factor. First, I tested the assumption of equal variance and found the variances were not equal. Thus, I performed a Welch test also. Race was coded as follows: 1=White, 2=Hispanic, 3=Black, 4=Asian, 5=Multicultural, 6=Other/Blank, 7=International. The results indicate there is no difference in pass rates based on race, $F(6, 5.96)=.76$, $p=.63$, but some race categories are likely too low to draw meaningful inferences. (see Table 12 for the descriptive statistics and Table 13 for the ANOVA results and Table 14 for the Levene's Test and Table 15 for the Welch test results).

Table 12

Descriptive Statistics for Pass Rates Based on Race (study group)

Race	N	Mean	Std. Deviation	Std. Error
White	46	.85	.36	.05
Hispanic	15	.93	.26	.07
Black	17	.76	.44	.11
Asian	3	.33	.58	.33

Multicultural	8	.63	.51	.18
Other/Blank	2	.50	.71	.50
International	2	.50	.71	.50
Total	93	.80	.41	.04

Table 13

ANOVA Source Table for Pass Rate Based on Race (study group)

ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1.65	6	.28	1.76	.12
Within Groups	13.47	86	.16		
Total	15.12	92			

Table 14

Levene's test for homogeneity of variances

Test of Homogeneity of Variances

		Levene Statistic	df1	df2	Sig.
Pass Rate	Based on Mean	3.24	6	86	.01
	Based on Median	1.20	6	86	.31
	Based on Median and with adjusted df	1.20	6	72.17	.31
	Based on trimmed mean	3.24	6	86	.01

Table 15

Welch test for equality of means (study group)

Robust Tests of Equality of Means

Pass Rate

	Statistic[a]	df1	df2	Sig.
Welch	.76	6	5.96	.63

a. Asymptotically F distributed.

While this pass rate shows promise, it is important to analyze the students that did not pass the course. There were six students in the study that earned a D, which allows the student to move to a beginning algebra course that is not credit bearing but counts as three-credits toward their credit load for the semester or a three-credit bearing quantitative reasoning course depending on their major. Thus, students that earn a D in this class do not have to retake the entire class. Of these six students, two had this class as the only class they earned a D or lower and they passed all their other courses. Two earned at least two other D's in the semester under study, one was a D in math and one in a different course, and 2 had a DFW in multiple courses. There were nine students that earned an F grade. Two of these students completed the course and took the final, and both failed multiple classes. The rest of the students with F grades completed 14 or less weeks of class. Two of these students passed their remaining classes, one had a D in a second class, and six failed multiple classes. Four students withdrew from the class, three of these students passed the rest of their classes and one failed multiple classes. Thus, 21 students earned a D, F, or W in this course and 14 did not pass at least one course other than their math class.

Since this course is only developmental credits, and no credits for graduation it is important to analyze how the students performed the following semester. Below Table 16 summarizes the results:

Table 16

Summary of Pass Rates for Spring 2020 (study group)

Fall 2019	Spring 2020 Class	Total Enrolled	Number of Students that Passed	Percentage that Passed
Students that passed	Did not enroll	6	0	0
	Contemporary applications of mathematics	6	5	83
	Intermediate algebra	54	39	72
	Mathematical logic	1	1	100
	Math for elementary school teachers	2	1	50
	Business calculus	3	3	100
	Total	73	58	79
Students that did not pass	Did not enroll	11	0	0
	Repeated Same Class	3	1	33
	Co-Requisite Algebra	2	2	100
	Contemporary Applications of Mathematics	3	1	33
	Total	19	4	21

Students in the study had a pass rate of 58% for their next math class in the spring. This does not give the entire picture because there were 17 students that did not enroll in school or a math class in the spring. It is unknown why these students did not enroll. Thus, looking only at the students that did enroll in a math class and were able to continue to their credit bearing math class, 79% passed their next math class. Additionally, three students took intermediate algebra over a three-week term between fall and spring and then proceeded to pass Business Calculus in the spring. Last, the pass rates for Spring 2020 based on race were not significantly different from each other $F(5,85)=.56$, $p=.73$ (see Table 17).

Table 17

Analysis of Spring Pass Rates Based on Race (study group)

ANOVA

S20Pass

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	.70	5	.14	.56	.73
Within Groups	21.43	85	.25		
Total	22.13	90			

Next I will compare the pass rates from the students in the study to students that took the course the last time it was run in a different format. The data collected for the historic data was from Fall 2010 through Spring 2013. I had to use a long period of time because there were very few students that took the course in those semesters. Unfortunately, because there were very few students it was difficult to obtain aggregate data. I was able to obtain how students progressed from the course to their next algebra course. Students in the study progress from the study course to the next algebra course and passed at a rate of 72%. The students from the Fall semesters of 2010, 2011, and 2012 progressed to their next algebra course and passed at a rate of 41%. Thus, the students in the study were better prepared to progress to their next algebra class compared to students in the previous class format.

Additionally, I compared how students in the study performed in their credit-bearing gateway course to the students that took that gateway course in Fall 2019 and Spring 2019. I was unable to obtain the Spring 2020 data as there was a delay in grade reporting due to COVID-19. Looking at the results, students that took Intermediate Algebra from the study passed at a rate of 72%. All students from Fall 2019 passed at a rate of 67%, and all students from Spring 2019 passed at a rate of 59%. Similarly, students that took Contemporary Applications of Mathematics from the study passed at a rate of 83%. All students from Fall 2019 passed at a rate of 81% and all students from Spring 2019 passed at a rate of 78%. Both of these results show that students passed their gateway course at least as well as students that were placed into the gateway course.

The results from the learning outcomes proficiency and pass rates is promising. Students show a greater understanding of fractions, integers and their operations after they have completed the course. In addition, students in the study passed the course at a rate of 81%. Looking at the students that went on to take their gateway mathematics course they passed that at least as well as students that were referred to the course. Last, the overall pass rate of students in the study compared to students in Tennessee and Georgia was at least the same. Next I will go on to discuss changes in students' attitude toward mathematics.

5.2.2 Research Question 2: How do the design changes affect students' attitude toward mathematics, and is there a change in discourse habits over the course of the semester?

This question was assessed using a quantitative analysis for the students' attitude toward mathematics and using a basic qualitative analysis to determine the discourse habits over the course of the semester. First, I analyzed the pre- and post-survey scores of the Attitude Towards Mathematics Index (ATMI). I received permission from Tapia (1996) to use the ATMI for research purposes. In our correspondence she indicated which questions were reverse coded and what questions corresponded to each of the different sub-scales. I proceeded to assign a point value of 5 for "Strongly Agree" and 1 for "Strongly Disagree". To determine if there was a change in students' attitude over the course of the semester, I conducted a paired t-test on the total score of the ATMI and each of the different sub-scales. The maximum score for ATMI is 200, the maximum score for each of the subscales is self-confidence (75), value of mathematics (50), enjoyment of mathematics (50), and motivation (25). See Table 18 for the descriptive statistics for the pre- and post-tests of the total score and each sub-scale. The analysis showed there is not a statically significant difference on the total attitude score, $t(57)=.44$, $p=.67$ (see Table 19). Moving attitudes is a long process for many individuals. It makes sense that there was not a significant difference in math attitude. There was a significant increase in students' self-confidence in their mathematics abilities $t(57)=3.69$, $p<.001$, $d=.48$. This indicates that the course does have an impact on a student's belief in their ability to do mathematics. This is evidence that some of the course components did promote a positive attitude change in students' mathematical mindset the premise of design principle 3. There was a statistically significant decrease in students' perception of the value of mathematics ($t(57)=-4.06$, $p<.001$, $d=.53$) and motivation towards mathematics ($t(57)=-2.00$, $p=.05$, $d=.26$). These are conflicting results may be due to the fact that the survey was administered at the end of the semester in a high stress period of their semester. In addition, we put these students in a state of disequilibrium. Because we asked them to be uncomfortable, it may lead to a decreased perceived value in mathematics.

Table 18

ATMI descriptive statistics

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Total post-test (200)	112.81	58	28.40	3.73

	Total pre-test	111.69	58	24.44	3.21
Pair 2	Self-confidence post-test (75)	40.79	58	12.32	1.62
	Self-confidence pre-test	35.76	58	12.35	1.62
Pair 3	Value post-test (50)	33.40	58	7.75	1.02
	Value pre-test	36.52	58	6.35	.83
Pair 4	Enjoyment post-test (50)	26.34	58	8.30	1.09
	Enjoyment pre-test	26.41	58	7.12	.94
Pair 5	Motivation post-test (25)	12.28	58	4.46	.59
	Motivation pre-test	13.00	58	4.09	.54

Table 19

Summary of ATMI comparisons

		Paired Samples Test							
		Paired Differences							
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	Total	1.12	19.62	2.58	-4.04	6.28	.44	57	.67
Pair 2	Self-confidence	5.03	10.38	1.36	2.31	7.76	3.69	57	.000***
Pair 3	Value of mathematics	-3.12	5.86	.77	-4.66	-1.58	4.06	57	.000***
Pair 4	Enjoyment of mathematics	-.07	6.21	.82	-1.70	1.56	-.09	57	.93
Pair 5	Motivation	-.72	2.76	.36	-1.45	.001	2.00	57	.05

***p<.001

The second part of this question is about the change in discourse habits throughout the semester. I observed each of the classes during a class that was talking about modeling rational numbers and plotting rational numbers on a number line. At this point in the semester, all the classes except one had students going to the board to put up their work. The instructor that was reluctant to have students come to the board solicited responses from students throughout the class. After the work was put on the board, each of the instructors went through and discussed the work that was on the board. All of the instructors had the students actively involved in discussing the workbook problems. When the students broke up into groups to work on the application problems, students stayed on task and talked about the problems. One instructor was walking around helping and a group was confused. The instructor provided guiding questions instead of providing direct answers. All the instructors left a good amount of wait time for student responses. All of the classes had student interaction throughout the class period. In one of the focus groups a student mentioned, “a lot of the times, like, it was us up on the board and, like, helping each other or showing each other.” She went on to say, “that was helpful, because sometimes you need somebody on your own level.” The student interaction was evident when one student talked in the focus group about her experiences in this math class compared to previous math classes.

> So, it wasn't -- I feel like, in all my other math classes, if I had something wrong I wouldn't say anything, because I knew the teacher would be, like, okay, you had this problem wrong. So, like, what would you do here? What would be the next step? And, like, [my current instructor] made it a group effort. So, it made people want to communicate more.

Another student in the focus group mentioned that “with our class, it was a little bit more, like, I wasn’t afraid to make mistakes.” Students responded well to the in-class discussion that took place. This discourse is evidence of the design features implemented in Design Principle 1.

When analyzing the student survey responses and the focus group responses two interesting themes emerged. The first theme that emerged is that students reached out to each other during class and outside of class. One student mentioned that “after class people message you” and ask for help with problems. Another student mentioned that “our whole back row has each other’s phone number. So, we all text each other.” In addition to the students helping each other, they are also holding each other accountable. In one of the focus groups there was a discussion about students that came to class without watching the videos or trying the homework. Both the students in the focus group were reluctant to help the student that did not do their work. In the survey responses, students also mentioned that they relied on their fellow students to work on problems together. The collaborative environment that existed inside the class continued outside the classroom walls. This is evidence of Design Principle 1: creating an equitable environment where students are comfortable participating in mathematical discourse.

The second theme that emerged was the support that students received from their instructors. Students felt that their instructor really cared about their learning. One student mentioned that "[the instructor] cares just as much about us understanding, as we care about need to understand it." The students seemed to understand the importance of reaching out to their instructors for help when they were struggling. There were eight students that mentioned one of the beneficial aspects of the course was reaching out to their instructors during office hours. This helped students make a connection with their teacher which is evidence of creating an equitable environment where students are comfortable participating in mathematical discourse. Next, I will go on to discuss the student perceptions of the course design.

5.2.3 Research Question 3: What are the students' perceptions of the course design?

5.2.3.1 What features of the course design do students believe is most important for their learning. To answer this question, I will combine the quantitative survey results with the qualitative short answer responses and responses from the focus groups. When coding the data, I looked for mentions of the 13 different course design features in the short answer responses that students were asked to rank. Students were asked to explain one of the three items they chose as most important to their learning and explain one of the three items they chose as least important to their learning (see Appendix B for the Student Impression Survey). In analyzing the question, it is not only necessary to look at what students found most important, it is also imperative to look at what students did not find important. Students will do work that they find important to their learning, and when they do not see the benefit to what they are doing they lose motivation to do the work. Thus, determining the design features that students do not feel are important will help either modifying the design feature or facilitate informing the students why the feature is important.

The median rank for each design feature is listed in Table 20. I performed a Wilcoxon analysis on each pair of design features and a False Discovery Rate test on each of the p-values from the results from the Wilcoxon analysis (see Table 21 – the p-values for each of the pairs are listed and the pairs that showed no significant difference are highlighted in green). The results showed that students ranked mandatory attendance and in-class activities with no significant difference and most important to their learning with a median rank of 3. Correcting mistakes and exam retakes showed no significant difference and important to their learning with a median rank of 5. Warm-up activities (median rank 6) and online homework (median rank 8) were ranked next with no significant difference. Next the group of grading system (median rank 8), order of topics (median rank 8), class discussion (median rank 8), mandatory discussion (median rank 9), and watching videos (median rank 9) showed no significant difference between any of the pairs. Last, study skills instruction (median rank 9) and TED talks (median rank 12) were ranked significantly different from all other design features.

Table 20

Summary of rank results for student perception of most important design features

	Median	Mean	Std. Deviation
Mandatory attendance	3.00	3.86	3.18
In-class activities	3.00	4.02	2.80
Correcting mistakes	5.00	5.08	2.27
Exam retakes	5.00	5.16	2.83
Warm-up activities	6.00	6.61	3.52
Online homework	8.00	7.27	3.75
Grading system	8.00	7.43	2.75
Order of topics	8.00	7.55	3.69
Class discussion	8.00	7.55	3.29
Mandatory discussion	9.00	7.90	3.80
Watching videos	9.00	8.18	3.90
Study skills instruction	9.00	8.94	2.68
Ted talks	12.00	11.45	2.42

Table 21

Analysis of design features ranking using Wilcoxon test and false rate discovery adjustment

	Attendance	In-class activities	Correcting mistakes	Exam retakes	Warm-up activities	Online HW	Grading	Order of topics	Class discussion	Discussion	Videos	Study skills
In-class activities	0.76											
Correcting mistakes	0.04	0.03										
Exam retakes	0.09	0.02	0.81									
Warm-up activities	0.00	0.00	0.02	0.03								
Online HW	0.00	0.00	0.00	0.01	0.49							
Grading system	0.00	0.00	0.00	0.00	0.20	0.67						
Order of topics	0.00	0.00	0.00	0.00	0.20	0.80	0.32					
Class discussion	0.00	0.00	0.00	0.00	0.25	0.59	0.85	0.84				
Discussion	0.00	0.00	0.00	0.00	0.17	0.49	0.63	0.60	0.58			
Videos	0.00	0.00	0.00	0.00	0.06	0.27	0.36	0.46	0.42	0.60		
Study skills	0.00	0.00	0.00	0.00	0.00	0.03	0.01	0.10	0.07	0.17	0.29	
Ted talks	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

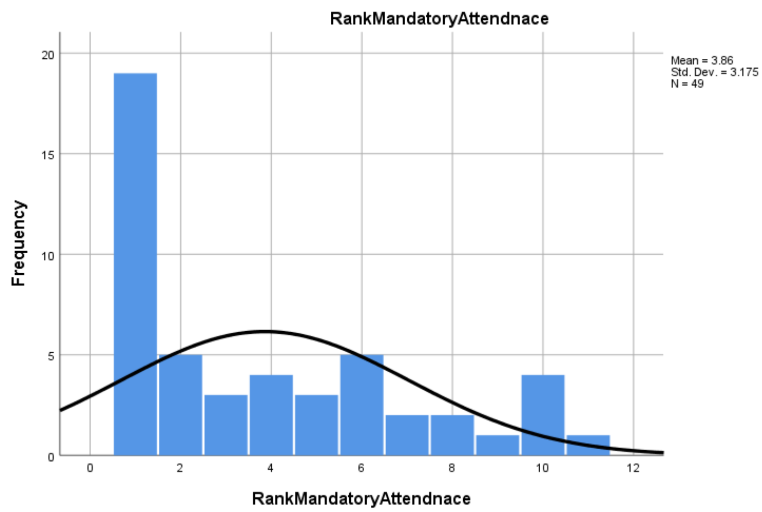
I will now proceed through each of the design features and describe the feature, indicate how the students ranked the feature and provide an interpretation of the short answer responses of each of the features. The features will be ordered from what students found most important to what students found least important.

5.2.3.1.1 Mandatory Attendance. In this course there is an emphasis placed on learning. Thus, one of the most important ways to learn is by attending class. So, not only is attendance mandatory for this class, students obtain grade points for active participation. The grade is based on students actively participating in daily classwork, group work, and formative assessments in class. Class participation encompasses 15% of their final grade (Design Principle 4).

In the survey results, mandatory attendance was ranked the highest, along with in-class activities, with a median rank of 3. Looking at the histogram (see Figure 8), the results are more convincing. Students felt mandatory attendance was extremely important to their learning. This category had by far the most first place vote of any other category.

Figure 8

Mandatory Attendance Histogram



Students had different reasons for why mandatory attendance was important. First, mandatory attendance was important because it was mandatory, and the students would lose points if they did not attend class. Second, students commented that you have to be in class to learn. One student stated "you actually get to go over the problems in class which helps you better understand the material." The third theme that came out was that if students miss class they fall behind. One student stated, "if you are not there then you will fall behind and soon lose motivation to get homework done and return to class."

Students understand that being in class is important to their learning. If they are not in class, they miss valuable learning opportunities. By having rich in-class activities discussed next, students see the reason to go to class.

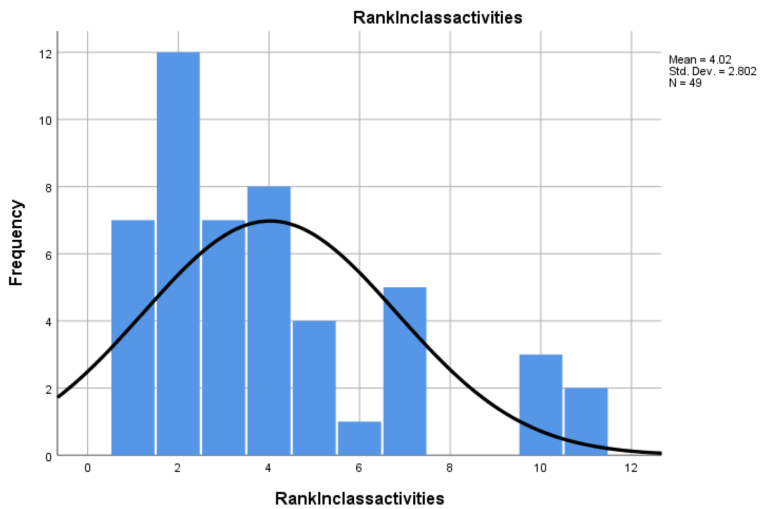
5.2.3.1.2 In-Class Activities. The in-class activities vary from day to day. Throughout the semester there are specific activities designed to help students gain a conceptual understanding of the material. Some of these include a paper folding activity that helps students manipulate equivalent fractions, some work with algebra tiles to show properties of real numbers, integer addition and multiplication, and some work with factoring (see Appendix H for examples of the in-class activities).

In addition to the targeted activities, students work on their workbook sections (see Appendix G for a sample workbook section) every day in class. Each workbook section is comprised of video notes, exercises, applications, discussion questions, and a review. The video notes correspond to the videos that students are expected to watch before class. The exercises correspond with what is on the videos and students should be able to do most of the exercises without support if they watch the video, though some exercises are slightly more challenging than what is demonstrated in the video. They are meant to challenge the students by incorporating prior learning into what they are currently learning. The application section has questions that have some real-world examples and extensions of the exercises (Design Principle 2). The discussion section asks conceptual questions to help students make connections between topics. Students are expected to complete the video notes and attempt the exercises before class every day. In-class the instructor chooses a student or a group of students to put the exercises on the board. Then, the solutions are discussed as a class (Design Principle 1). After all the exercises have been discussed the students split into groups and work on the applications. The instructor walks around and helps the groups that need help. Finally, when it appears the groups are wrapping up, there is a class discussion about what came up in the applications and then the workbook discussion questions are talked about. Workbook sections range from one day to four days of work depending on the topics discussed.

The in-class activities also had a median rank of 3. Looking at the histogram (see Figure 9) it was important for most students' learning.

Figure 9

In-Class Activities Histogram



While students ranked the in-class activities relatively high, not many explained why they ranked it high. Of the few that did discuss their ranking some indicated that the in-class activities were important "because without them I wouldn't know how to do some of the math since we basically had to teach ourselves at home with the videos." Other students felt it was important going through the workbooks in class also. It is interesting that some students saw the before class work as completely teaching themselves, as opposed to the beginning of the learning process, while others thought it was beneficial to see how others approached the problems. In fact, in the focus group one student explained that in class there was many times they were learning from other students, and also teaching other students. They go on to say "you are still learning while you're teaching people, which is cool." This same student mentioned, “I think that doing it in class and having so much time to throw us up there and make us kind of teach it, really helped me with the way that I process information. Versus, when I’m left to my own devices, I get kind of lost.”

There were a couple students that did not feel the active learning portions of the class were beneficial to their learning. Some felt it would be better to work as a whole class instead of groups/individual. Others felt that learning from a lecture and doing homework at home would have been more beneficial. This class can be uncomfortable for students as it does hold students accountable for their own learning. Students that have not experienced this type of learning and also struggle with the mathematics can experience discomfort. Usually this discomfort can be alleviated with continued support from the instructor.

Last, the students that commented on the conceptual learning activities felt that sometimes they were more confusing than helpful. Some of the conceptual learning activities can be confusing for some students, especially if they have been taught a procedural approach to mathematics. They are accustomed to a rule, as opposed to conceptually understanding why the rule works. When students try to apply the rule to the conceptual activities it can be confusing.

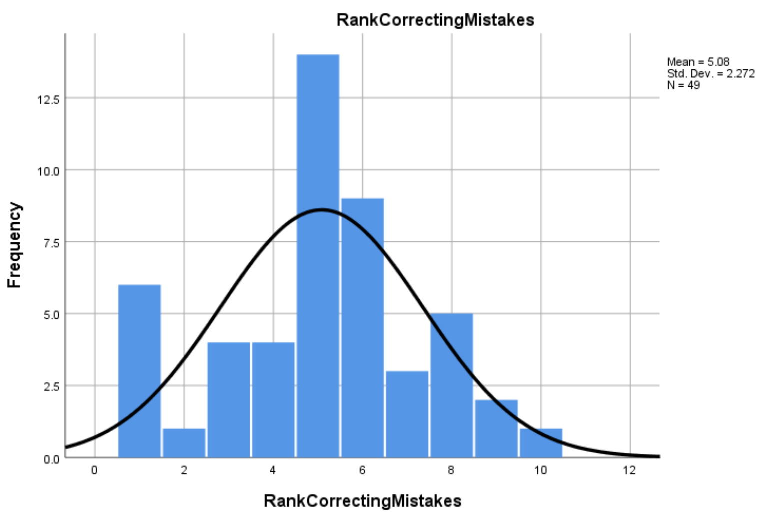
Students feel the in-class discussions are important to their learning. They perceive that the in-class work as separate from the work that is done before class, instead of an extension of that work. The instructors should ensure that they place the importance on the extension of the before class work into the in-class work when they are talking with their students. This will help students see the coherence of the workflow. Additionally, instructors need to support students when they are struggling with understanding why the course is structured as a flipped classroom instead of a lecture-based classroom (Design Principle 1). Last, instructors should let students know it is acceptable to be confused when they are working on conceptual learning activities (Design Principle 2). This struggle is evidence of their learning. If students are aware that the struggle is encouraged, they are more apt to take part in the activity (Design Principle 3). Next, I will discuss the analysis of correcting mistakes.

5.2.3.1.3 Correcting Mistakes. As discussed previously an important feature of this course is helping students learn from their mistakes. Therefore, in this class there is an emphasis placed on learning from mistakes. This helps students gain a conceptual understanding of the material (Design Principle 2) and provides students an opportunity to understand that because they made a mistake, they can learn from it (Design Principle 3). To this end, all written work turned in is given the opportunity to have corrections done. All workbook assignments are used as formative assessments and are revised until they have little to no errors. The workbooks are evaluated to make sure that students are writing everything mathematically correct. If there is work that has math grammar errors students will need to correct the error and re-submit the homework. All workbook assignments need to be complete and correct to earn eligibility to take an exam. All exams except the final are also required to be corrected regardless of the score they received on the exam. The exam corrections are a key assignment in the course and are assigned grade points, but the corrections do not change the actual exam grade. Last, during the lecture part of the class students put their own work on the board and then the work is reviewed as a class. Any work that has mistakes is discussed, and corrections are made. Students are encouraged to correct any of their in-class work as the class progresses.

The median ranking for Correcting Mistakes was 5, which showed no significant difference with exam retakes discussed below. Looking at the histogram (see Figure 10), it shows that the largest number of students ranked correcting mistakes in the middle of the beneficial to learning rankings.

Figure 10

Correcting Mistakes Histogram



Even though the majority of the students felt it was of medium importance to their learning, many chose it as one of the top three to explain why they felt it was important. The perception about correcting mistakes as that "it allows you to go back and fix the mistakes that you made and helps you learn from them." There was another student that mentioned, "I appreciate having the time to do that instead of just forgetting about it."

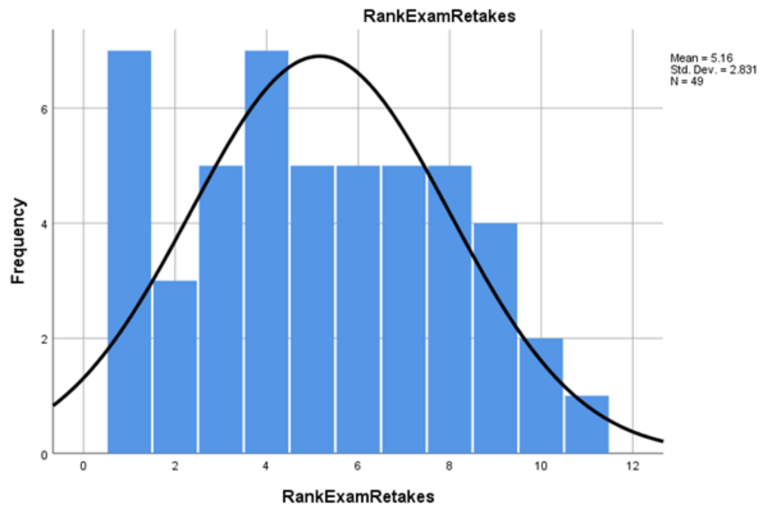
While many students perceived correcting mistakes was medium importance, when it was discussed students felt it was important. Students are able to learn what their misconceptions are, and how to correct those misconceptions so they can gain a conceptual understanding in the course. This helps them better prepare for summative exams, and subsequently prepares them for their next mathematics course. Next, I will discuss the analysis of exam retakes.

5.2.3.1.4 Exam Retakes. Mathematics builds on itself. Thus, if students have a firm foundation with very few misconceptions or gaps, they will understand more of the course. Therefore, students are required to retake any exam (other than the final) if they earn less than an 80%. To be eligible to retake the exam they must turn in the corrections to the original exam and meet with their instructor. Usually if a student earned between 70% and 80%, they either do a partial retake or an oral retake. Students in this range usually only have a few problems they didn't know how to do and did well on the rest of the problems. Students that earned less than 70% have to do a full retake. All retakes are capped at an 80%, so students don't purposefully do poorly on the original exam just to be able to retake it.

The median ranking for Exam Retakes was also 5. Looking at the histogram (see Figure 11), it shows that students ranked exam retakes in almost every position.

Figure 11

Exam Retakes Histogram



Interestingly, even though the rankings are even among all the ranks, 18 people chose exam retakes as one of the top three items they wanted to describe. The perception is that the exam retakes are important to students' learning. One student stated, "exam retakes are one of the best things that can happen to a student, it's like a second chance." This was the sentiment from many students. One student in the focus group stated that the exam retakes were important because it provided a second opportunity even though they never had to use it. Another student in the survey stated that exam retakes "kept me motivated." Thus, by providing exam retakes students were able to not give up if they did poorly on an exam. They were able to figure out the mistakes they made or the gaps they had, re-learn the material and try again. The students perceived that this helped them stay motivated even with one bad grade.

Students understand that exam retakes are important to their learning if they need to use one. Many even saw the importance of the retake even though they never earned under an

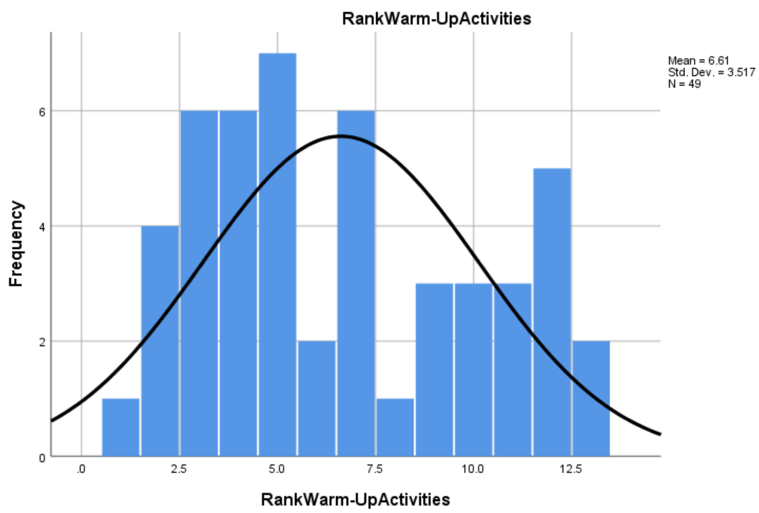
80% on an exam. Allowing exam retakes shows the students that the course places an importance on student learning and not a summative grade at the end of the semester (Design Principle 4). In addition, since the students have to complete the corrections on their original exam, students are able to learn from their mistakes (Design Principle 3) and gain a better understanding of the material (Design Principle 2).

5.2.3.1.5 Warm-up Activities. The warm-up activities are designed to be quick number talks that are accessible to all students. They promote engagement and discussion in the class. They help students not be nervous to talk in class. The warm-up activities are one of the aspects of the course that I model in the initial course training, and we talk about in our first few course meetings. I do leave it up to the instructors if they want to do the number talk or not. If the instructor is not comfortable, they usually will not do the number talks. The number talks for most of the sections of the course stopped after the first couple weeks this semester. There were a number of factors including instructor comfortability and scheduling that contributed to them being stopped.

The warm-up activities had a median ranking of 6, which had no significant difference with online homework. The histogram shows the reaction is mostly favorable rankings, but steady among all the ranking (see Figure 12).

Figure 12

Warm-Up Activities Histogram



There were very few comments on the warm-up activities in the short answer questions. Students understood that they were designed to get their brains awake and ready to learn, but the students that commented did not find them useful or relevant to the material that was being discussed in class.

The warm-up activities have to be related to what is being discussed in the course. Students do not see them as beneficial if they are not related to the material being discussed that particular day. The warm-up activities are very beneficial the first few days of class. It enables students to open up and be comfortable talking in a math class. Students are typically reluctant to talk if they believe they have an incorrect answer. Using warm-up activities, the first weeks of class helps students see there is sometimes many different ways to interpret problems, and even if the answer is incorrect it is still a learning experience. Next, I will discuss the analysis of the online homework.

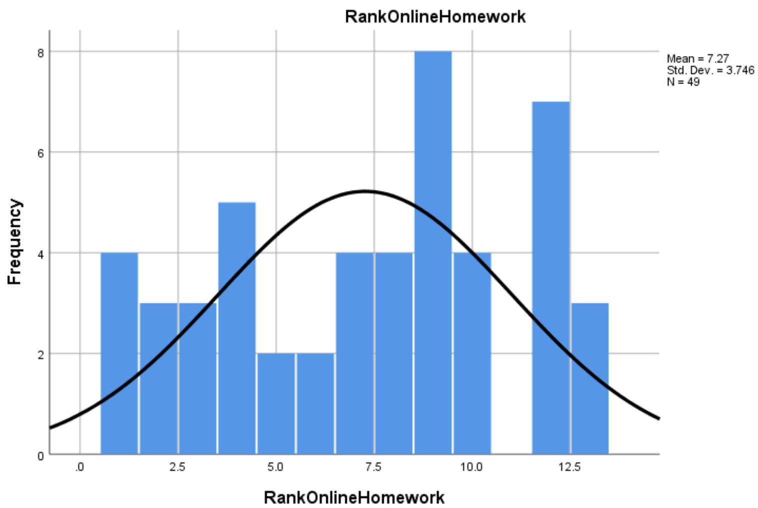
5.2.3.1.6 Online Homework. The online homework is designed to supplement the in-class work and workbooks by providing extra practice for students. There were two different online homework programs used as a supplement to in-class work this semester. All sections but one used ALEKS, which is a mastery-based program that bases its learning path on an initial knowledge check and then periodic knowledge checks throughout the semester. Students start learning after the initial knowledge check and then after they finish objectives, they are given another knowledge check to make sure they retain what they are learning. If students answer topics correctly on the knowledge check that they haven't learned, ALEKS adds it to their mastery. If students answer topics incorrectly, the topics are put back for the students to re-learn. If students have little prior knowledge, struggle with a particular topic, or rush through a knowledge check it is easy for the student to get behind in their learning path. Thus, the learning path a student is on may not correspond to what a student is learning in class if they are too far behind.

One section piloted Knewton Alta for the semester. Knewton Alta is also a mastery-based program, but instead of assessing a student's knowledge and then building a learning tree from there, Knewton Alta assumes that a student has the prior knowledge of the assigned work and remediates back if the student is struggling. Since the students' learning path is not based on a knowledge check, students were able to stay on track with the material. In addition, the learning path they are on mirrors what is being talked about in the class sessions.

The survey results for online homework were mixed to low. The median ranking for online homework was 8. The histogram shows that there is mixed reaction to the online homework (see Figure 13).

Figure 13

Online Homework Histogram



In the short answer questions and focus group most of the comments were negative. Students felt the "online homework was more of a hassle than anything," and "ALEKS should not be a requirement". Students were concerned about the amount of work ALEKS required and because it was a requirement it prevented them from taking an exam. One student mentioned that ALEKS is "really redundant and it brought my grade down more than the beneficial aspects to the class."

Conversely, there were some students that felt they learned from ALEKS. One student stated "the most beneficial part is the online homework (because it shows step by step how to do something if you got it wrong)." So, for some students the online homework helped them learn instead of posing a barrier to their learning.

Some students viewed ALEKS as a barrier to their learning. ALEKS in particular is more procedural and therefore does not always match the conceptual goals for the course. Additionally, ALEKS is a mastery-learning program that is based on periodic knowledge checks.

Thus, if a student forgets topics it is very possible for the student to be required to re-learn topics before they are allowed by the program to progress to the current topics. There were not any explicit comments on Knewton Alta, so when the online homework was mentioned it may include both of the platforms. The students mentioned that the online homework should not be a requirement for the exams. This requirement has always been in place since the course has been in this format. Since the online homework is designed as an extension of the material that is done in class to help students gain a better understanding and fluency with the material (Design Principle 2), I would hesitate to not make it a requirement. Most students will only do the required work for a course even if the optional work will greatly help their understanding. To that end, I will discuss the analysis of the grading system next.

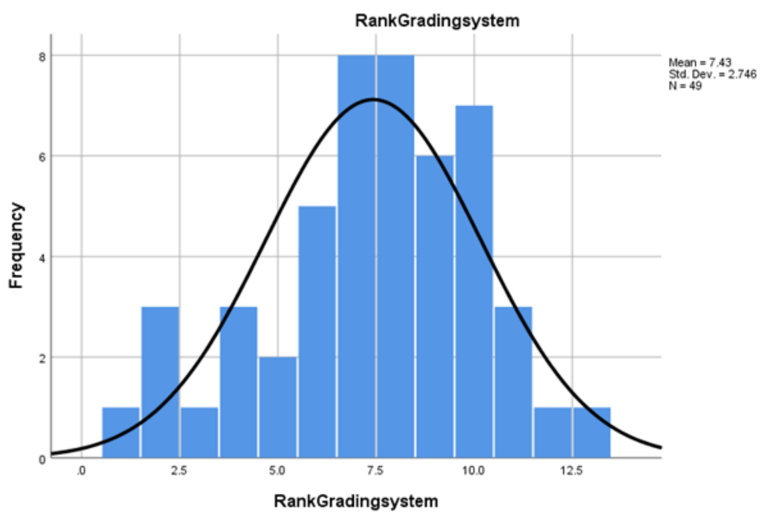
5.2.3.1.7 Grading System. The grading system in this class is constructed for students to focus on learning (Design Principle 4).Boaler (2015) discusses the importance of learning from mistakes. In addition Weimer (2013) discusses the importance of placing grade points on the work that the instructor deems important for student learning. Thus, the grading system in this course includes points for daily work and coming to class, as well as exams. As you can see in the syllabus (see Appendix J), most graded items are around 10% of their final grade. This places the importance on doing all of the work in the course, not just high-stakes tests.

One aspect of the grading system for the class is that students have to qualify to take exams. Students need to be completely caught up on their homework (written and online) and complete the exam review. This can be beneficial to the students that do stay caught up with their homework, and hard for students that do not. Students are reminded regularly of the work they need to complete and are encouraged to get help from the instructor or tutoring if they need it. If students do not qualify for the exam, they are able to complete the work and take the exam the following week. This is counted as an exam retake.

Recall, the ranking for grading system, order of topics, class discussion, mandatory discussion and watching videos showed no significant difference. Looking at the quantitative results of the ranking, the rank of the grading system had a median rank of 8. Looking at the histogram, many students ranked it in the middle between 6 and 10 (see Figure 14).

Figure 14

Grading System Histogram



Therefore, there were not many comments on the grading system in the short answer questions. The students that did comment on the grading system felt the grading system was important because they could see how things were graded and knew what to do to obtain a high grade, or they felt the grading system was harsh. Looking a little more in depth at the students that felt the grading system was harsh, the criticism was made in conjunction with the online homework being an exam requirement. The online homework for all but one of the sections was mastery based and based on an initial assessment. If the student fell behind or did not perform well on a performance assessment it was difficult for the student to catch up with the material. Therefore, it was more difficult for students to qualify for exams. One student stated that online homework was one of their bottom three because, “I just felt like it was a burden and a stress creator. I didn’t want to do it but if I didn’t then I couldn’t take the exam.”

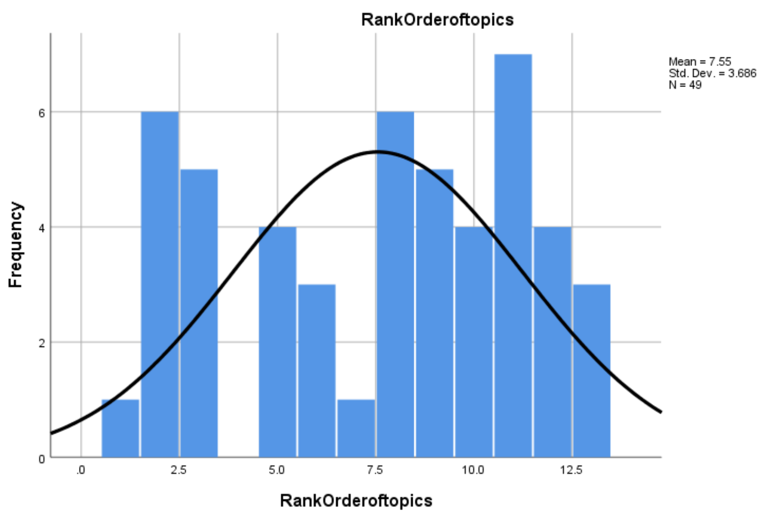
The grading system seems to be an invisible design feature of the course. Many students did not even comment on the grading system. The comments that were made, involved qualifying for exams and the stress it created. The students that qualify for the exam perform better on the exam. We know that students have many other requirements on their time in addition to this course. Thus, it can be easy to fall behind or procrastinate on doing the homework for courses. Since all the work contributes to the grade and is required to be eligible to take exams, students cannot put the work off completely. Therefore, it can be stressful if they are trying to complete the work to qualify for exams and had put too much work off for later. The importance of this policy is evident in the success rate of the class though. With almost 80% of the students passing, requiring students to do their work is important. Next, I will discuss the analysis of the ordering of the topics.

5.2.3.1.8 Ordering of topics. The order of the topics for this course is different than a traditional developmental mathematics class (see Appendix E). In a traditional developmental mathematics class, the topic order follows a similar progression as K-12 except it is usually accelerated. This course combines many of the topics into operation groups so that students gain a conceptual understanding.

The median ranking of the order of the topics was 8. Looking at the histogram, there was not a consensus on the ranking for order of topics (see Figure 15).

Figure 15

Ordering of Topics Histogram



There were only two comments on the order of topics for the course in the short answer portions. Neither described why the order was important to their learning, they just stated the order was important. In the focus groups when the question about order of the topics was asked, the participants focused on the individual topics instead of the bigger picture of how all the topics worked together. It seems that this may have been one of the invisible aspects of the course design, where students were unaware of the differences because the topics fit together.

It is difficult to tease out the importance of the ordering of the topics. The design feature is invisible to students since most have never taken a different developmental mathematics course. Students tend to not put much thought into how topics in the course are ordered. If the course content has continuity and they see topics as following one to the next they usually do not realize the content was ordered differently than other developmental mathematics courses. In fact, when the question of ordering was brought up in the focus group the students began talking about a specific topic of instruction, not the course as a whole. Because the students have never experienced something different, they do not know the benefits or drawbacks of how the topics are ordered. Next, I will analyze student perceptions of the discussion section.

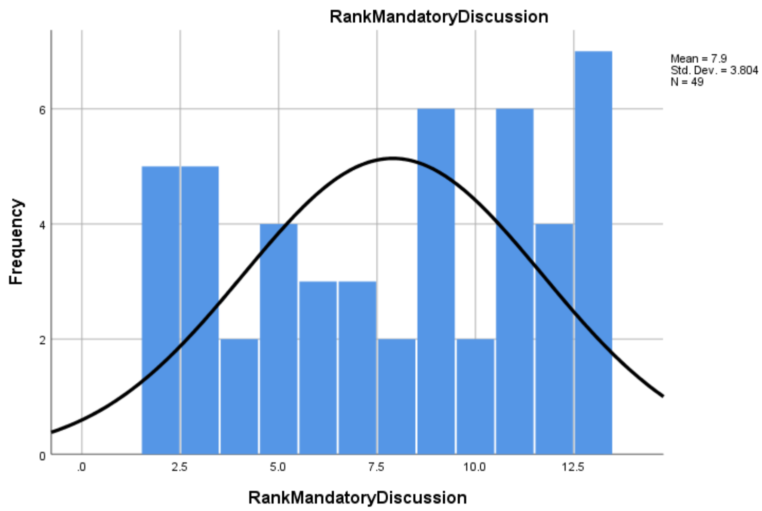
5.2.3.1.9 Discussion. Students taking the survey interpreted "Class Discussion" as the "Mandatory Discussion". This was determined after reading the survey short answer questions. The responses talked about the actions that take place in the discussion section, not during the class discussion that take place in the class session. Therefore, these two design features have been combined in the analysis.

Students in this class also signed up for a mandatory discussion section. The students in one discussion class did not correspond directly to a specific lecture section. Rather, in one discussion class there were students from each of the different lecture sections. Students signed up for the discussion section that fit their schedule best. The discussion portion of the class consisted of two different parts. During the first half of each discussion time the students would complete a written worksheet as a class. The first half of the semester the written worksheet was based on study skill videos watched as a class. The second part of the semester the written worksheet was based on various application problems that corresponded to the topics that were being discussed in the lecture sections. The application problems were chosen to be slightly harder than they received in their online homework, comparable to questions they were doing in the lecture workbook. The second half of each discussion time was devoted to completing their online homework. There were two different online homework platforms used this semester. One section piloted Knewton Alta and the other sections used ALEKS.

Student perception of the mandatory discussion was mixed. Looking at the quantitative results, mandatory discussion had a median ranking of 9. But, looking at the histogram of the rankings, they are mixed (see Figure 16).

Figure 16

Mandator Discussion Histogram



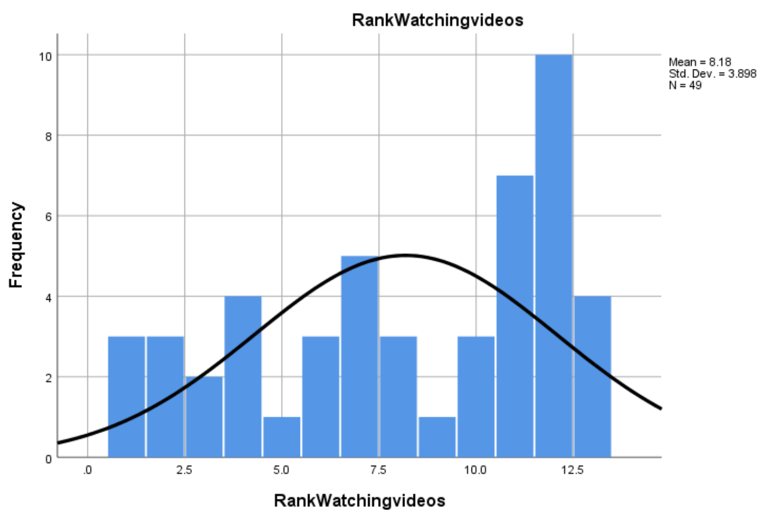
When analyzing the short answer responses, only one student answered that mandatory discussion was beneficial to their learning, saying it “gives me more time and help I need to succeed in this class”. The overwhelming consensus about mandatory discussion was that it was “pretty much a waste of time”. Many students felt that it should only be required if the students were struggling, and it was similar to a study hall. The students felt it wasn’t helpful and watching the study skills videos did not help them learn how to solve the math problems. When talking about the discussion in the focus group, one student mentioned that it would be better structured if the discussion section was set up similar to the active learning lecture section. The students felt they benefit from the interaction between their fellow students and would benefit from that more than just working on their online homework.

Therefore, students do not seem to see the benefit of completing their homework in an environment where there is support available when they need it. They feel this is work that they can do at home on their own time. This is interesting because the students in the focus group discussed how they had a group of students they worked with in the discussion section. This support emphasizes the aspects of Design Principle 1. This design feature may be an aspect of the course that students do not perceive as important but is very important as they are finishing their online homework assignments and learning valuable study skills. Next, I will discuss students' perception of watching videos before class to prepare for in-class learning.

5.2.3.1.10 Watching Videos. This class is set up as a flipped class. Students watch videos pertaining to the learning outcomes in the next day's class. While watching the video, students take notes and work on exercises based on the learning outcomes. The exercises are based on what was discussed in the video and an extension of some of the learning outcomes. One of their grade items is to watch the video, take notes and attempt the exercises before class. The median ranking of watching videos was 9. The histogram shows that many people felt it did not benefit their learning (see Figure 17).

Figure 17

Watching Videos Histogram



Looking at the short answer questions that mentioned watching videos many students believed that they would learn better from someone that was lecturing at them in the class. One student suggested that the one thing they would change would be "not doing the videos and actually teach the material in class." Another student mentioned that "I didn't find the videos very helpful when watching and taking the notes, I couldn't raise my hand and ask questions about parts I was confused on or ask the instructor to go back to a certain step." It seems that many students did not see the connection to the beginning of their learning by watching the video and then rounding out their learning with the in-class activities. The videos are very similar to how a lecture would run if they were in class.

There were some students that felt the videos were helpful. One student mentioned, "the before class videos are important to my learning because they help me understand my homework better when there isn't a lot of time to go over it in class." In the focus groups there was more time to discuss the different aspects of the course. We discussed the watching videos before class more in depth. The first focus group consisted of two students from different classes. The perspective of both students was that sometimes the exercises associated with the videos was more challenging than what was covered in the video, but if that happened, they realized they could understand it better in class. One student was frustrated with students that did not watch the video. They stated “I think a lot of the problem too with the kids that don’t understand that, really just don’t understand anything, is they don’t watch the videos.” In a different focus group, the students mentioned they would rather learn from a teacher, but one of the students said,

> I felt kind of similar at first about the videos and the online work. So, it’s kind of, like, I’m not going to do good if I have to, like, show myself at home. But, then, I said five, because I think that doing it in class and having so much time to, like, throw us up there and make us kind of teach it, really helped me with the way that I process information, versus, like, when I’m left to my own devices, like, I get kind of lost, I guess, so that was helpful.

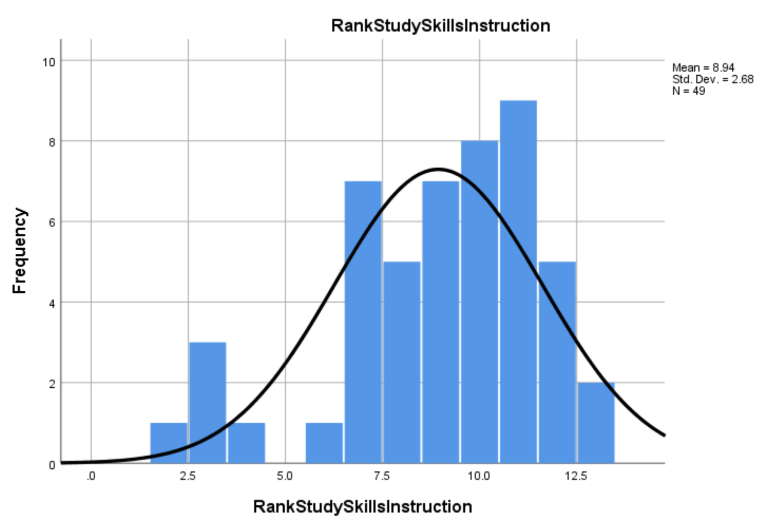
When introducing the class, the instructor talks about the flow of the class and the importance of how it all works together. This needs to be discussed at regular intervals throughout the course. Many students believed that the in-class activities were important to their learning, but about the same number believe that watching videos is not important to their learning. The design feature of providing a flipped classroom to facilitate rich in-class learning experiences needs to be emphasized. Students will place an importance on the activities they feel are most beneficial to their learning. If they realize the importance of watching the videos before class and see the benefits of the rich class discussions because everyone has come to class prepared, they will do what is necessary to succeed. Next, I will discuss the analysis of the study skills instruction.

5.2.3.1.11 Study Skills Instruction. Study skill instruction is provided to help students develop a better understanding of how to learn math (Design Principle 3). Study skill instruction was done in the discussion class. For the first half of the semester students would watch a short video that talked about notetaking, test-taking, time management, etc. at the beginning of the class session. Then they would get together in groups and fill in a worksheet based on the video. The worksheet would ask questions about the video and ask the student to demonstrate what they learned from the video. In addition, the workbook itself has a portion of the beginning of each section with a place for video notes. The video note section was similar to how the video lectures were organized.

Study skill instruction had a median ranking of 9. The histogram shows that the ranking is very skewed, compared to some of the other course design features (see Figure 18).

Figure 18

Study Skills Instruction Histogram



There was one person that thought the study skill instruction was beneficial in the short answer questions of the survey. This student stated, “this helps math is one of the subjects that are not easy to study for, so with this resource it helps us find different/easy ways to study for our exams.”

The other students perceived the study skills instruction as something they know how to do, and it was not teaching them how to do the homework. One student summed up the other responses, “Watching YouTube videos on how to be a student didn’t benefit anyone and didn’t teach me how to solve the math problem. It’s a nice idea but all of those videos were common sense. I’d rather actually learn how to do the homework instead.” Another student mentioned that they did not feel the study skills instruction benefited their learning because it “didn’t really help me change my study habits.” Thus, students perceived that they were getting the instruction from other places or they already knew what they should be doing. Even if they did learn something from the study skills instruction it didn’t help them change their study habits and they felt it took away time from their mathematics learning.

It seems many students do not see the importance placed on how to learn math. The videos talk about how to take notes in a math class, but do not talk about how to take notes in this specific class. The preparing for the exam videos generalized how to take a math test and did not talk about this specific course. Students seem to have trouble relating the generalized study skill instruction with how to apply it to a specific course. They believe that learning math is just doing the math problems. But students referred to this class struggle with how to learn math. Therefore, there should be an even greater emphasis placed on learning how to learn math. Some of the instruction is only done in discussion and then students are expected to be practicing the habits they learned throughout the course. There should be follow-through to ensure that students are continuing the practices they learn in discussion. Next, I will analyze students' perceptions of the TED Talks.

5.2.3.1.12 TED Talks. There were various TED talks that were played in lecture and discussion throughout the beginning of the semester. The TED talks were meant to provide students with an expert talking about a motivational idea related to math learning. Some of the TED talks included

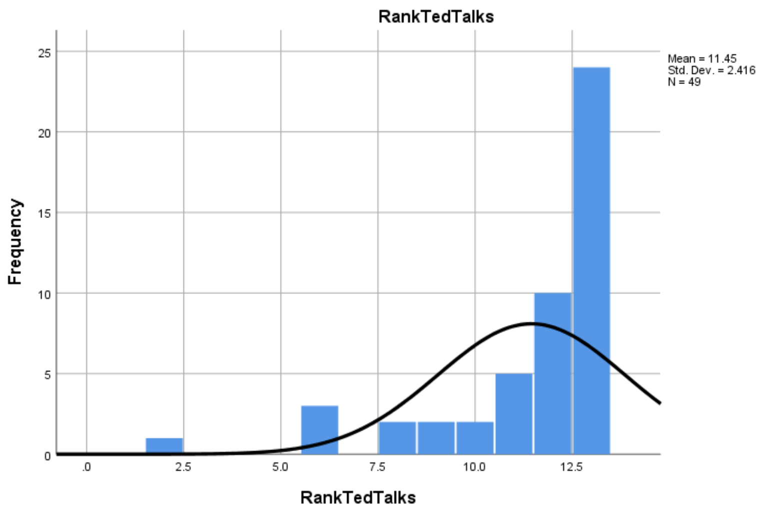
- Sal Khan – Let's teach for mastery
- Angela Duckworth – Grit
- Laura Vanderkam – How to gain control of your free time
- Kelly McGonigal – How to make stress your friend
- Roger Antonsen – Change your perspective

After watching the TED talk there would be a class discussion or a free write about the video.

In the survey results TED talks had a median ranking of 12. Looking at the histogram, most students did not feel the TED talks were beneficial to their learning (see Figure 19).

Figure 19

Ted Talks Histogram



Reading through the comments about the TED talks, students overwhelmingly did not feel that the TED talks supported their learning. Students felt that the TED talks were a waste of time and did not support their mathematics learning. Some students identified that the TED talks were supposed to help with their motivation, but they did not perceive that the TED talks were important. The students said, " the TED talks were more motivational and helped me want to do good in math although they did not teach me anything like the before class learning videos," and "TED talks are more motivational which is positive, but they do not really help me understand math better or provide time management skills that I didn't already know about." Many other students felt that the TED talks were "not helpful at all" and "added nothing to the course."

Many students feel that the TED Talks take away from instructional time that they could be learning the mathematics for the course. But, similar to the study skills instruction, helping students understand different aspects of the course design by listening to experts talk is beneficial to student learning (Design Principle 3). That being said, it is difficult to incorporate these into the classroom if the instructor struggles with opening up a class discussion about what was discussed in the video. This may be a design feature that is available to instructors if the feel they are able to facilitate a discussion of the video after it is over. Next, I will go on to discuss the overall results from the students' perceptions of the design features of the course.

5.2.3.1.13 Student Perception Summary. The students felt the most beneficial parts of the course are the ones they felt directly impacted their learning of mathematics. Mandatory attendance, participating in the in-class activities, correcting their own mistakes and learning from others' mistakes, and exam retakes had a direct impact on their grade, and they could see the impact it made in their learning. Therefore, students perceived these design features to be the most beneficial. TED Talks, study skills instructions, watching videos before class, and discussion were seen as not as beneficial. These design features had a more invisible benefit to students, and therefore students did not see the impact on their math instruction. Additionally, students come into the class with prior knowledge of how they have been taught math previously. Most students have not experienced a math class that is flipped, with an active learning classroom. Thus, some students believe that they struggle to learn math in this environment because they have not learned math in this type of learning environment before. Next, I will go on to discuss the perceptions of the learner-centered assessment system.

5.2.3.2 What are the students' perceptions of learner-centered assessment system? The learner-centered assessment system is multi-faceted for this course. The focus in the course is to provide formative assessment support to facilitate success in summative assessments. Thus, all daily work focuses on mastery and is allowed corrections until mastery is met. The two types of daily work that students turn in are their workbook sections and their online homework. The workbook sections are discussed thoroughly in class and then students are asked to turn in the work when they have completed the work. Some students turn in the workbook packets right away, and some need to take a couple days to complete the work. There is no specific deadline for the workbook sections other than the Monday before the exam. Students are required to have all workbook sections complete and correct to be eligible to take the exam. The purpose of this requirement is stated as why would you want to take an exam you are not prepared to take. By completing the workbook sections, students are more prepared to take the exam. In addition to having the workbook sections complete, they also need to be correct. The work is discussed in class, and students are allowed to make corrections on their work when we are discussing it in class. When the students turn the workbook sections in, the instructor looks over the work to ensure that the students are writing the mathematics correctly and their answers are correct. If the work has mistakes, the section is returned to the student and the student makes corrections where needed. Students are able to get help from the instructor if needed.

The second aspect of the daily work is the online homework. The online homework is mastery based, so struggling students may need to do more work if they are not understanding the topics. Depending on the system that is used that comes in different forms. See the online homework section for an in-depth description.

Last, students are required to meet eligibility requirements to take a summative exam in the class. The messaging is that the student will do their best if they do all the work that pertains to the exam. The exam eligibility consists of students completing all the workbook sections that will be assessed on the exam, completing all the online homework sections that will be assessed on the exam, and completing their exam review. Then, if a student was not eligible to take the exam, they are allowed to become eligible and take the exam the following week. In addition, if a student scored less than 80% on the exam, they are required to take the exam a second time. This can be an oral exam in the instructor's office, a partial retake if the student didn't show mastery on a small number of problems, or a full retake. If a student needs to take a retake, they are required to make corrections on their exam first.

The Student Impression Survey (see Appendix B) has multiple Likert questions that address the importance of the assessment system. These questions include

- The worksheet corrections helped strengthen my understanding of mathematics
- Knowing that I have to qualify to take exams better prepares me for the exams
- I feel there is an emphasis placed on learning in this course
- Self-correcting my mistakes helps me understand my misconceptions
- I feel the grading system values my learning
- I feel that all the work I complete in the course has value
- I am glad I have the ability to re-do an exam if I do not perform well the first time

For each of the responses, I coded each of the responses 4 = "Strongly Agree", 3 = "Agree, 2 = "Disagree", and 1 = "Strongly Disagree". Looking at the analysis, students perceive that all aspects of the learner-centered assessment system were beneficial to their learning. All of the students that responded to the survey either agreed or strongly agreed that they were glad for the ability to have a retake if they needed it. All of the categories had less than 10 students disagreeing with the statement except for "Knowing that I have to qualify to take exams better prepares me for the exams," and "I feel that all work I complete in the course has a value." See Table 22 for a summary of the perception of the assessment system

Table 22

Summary Table for the Perception of the Assessment System

	Strongly Agree/Agree	Strongly Disagree/Disagree
The worksheet corrections helped strengthen my understanding of mathematics.	51	6
Knowing that I have to qualify to take exams better prepares me for the exams.	42	14
I feel there is an emphasis placed on learning in this course.	50	6
Self-correcting my mistakes help me understand my misconceptions.	49	7
I feel the grading system values my learning	47	9
I feel that all work I complete in the course has a value.	37	19
I am glad I have the ability to re-do an exam if I do not perform well the first time.	56	0

The majority of students feel that all aspects of the learner-centered assessment are beneficial to their learning. The items that had the most disagreement included having to qualify to take an exam and that all the work has value. This is consistent with the student perceptions of the design features that were least visible to their learning. If students do not see an importance to the study skills instruction and online homework, they may not perceive that it has a value to their exam performance or the course as a whole.

In addition to the Likert scale questions mentioned above, there were responses from the Student Impression Survey and the focus groups that provided insight into the student perception of the learner-centered assessment system. Looking at the perceptions of students that related to these aspects of the course, I looked for when students mentioned corrections or correcting mistakes, exam retakes, and exam eligibility. Students perceived both correcting mistakes and exam retakes as beneficial to their learning. Many students mentioned that when correcting mistakes, they were able to “reflect upon what I did wrong and learn from that mistake.” In the focus group one student mentioned that they were given all their workbook sections back for corrections a day or two before the upcoming exam. This was not how the course was intended to progress. Instructors were supposed to grade and return homework within a day or two of receipt. The student in this class expressed that this was similar to cramming and did not facilitate their learning.

Students that mentioned the exam retakes felt they were beneficial to their learning. This included the students that did not use the exam retakes. They felt that by just having a safety net it lowered the stress of the actual exam. One student mentioned, “If I didn’t pass the first time, I could always get a better grade next week and it won’t affect my score much.”

There were no overall comments on the course policy of exam eligibility either in favor or against. There were comments on online homework being a requirement for taking an exam. As discussed before, if a student falls behind in their online homework it can be extremely difficult to get caught up. Therefore, students that put off doing their online homework would struggle to complete the work because it was different topics than were being discussed in class, and there would be quite a few topics to complete in a short period of time. Thus, students perceived that online homework should not be a requirement to take exam. One student summed it up as, “ALEKS is fine as practice work but not as a requirement. That also leads to the grading system that mandates ALEKS. Students struggle to keep up because this is too much work on top of work.”

The learner-centered assessment system as a whole is not the most visible aspect to the course. Students may not realize that there are other ways to be assessed in different courses, or they may accept that this is the way the course is assessed, and they cannot change it. There are a couple features of the assessment system that students do not perceive as beneficial to their learning, including online homework, and others that they perceive as beneficial, for instance exam retakes. Overall, the students agreed that the design features that are associated with the assessment helped them in the course. In the next section I will discuss students' perceptions of the opportunities to promote a growth mindset.

5.2.3.3 What are students' perceptions of the learning opportunities designed to promote a growth mindset? Many of the learning opportunities designed to promote a growth mindset towards mathematics are designed to be invisible to the students. Some of the learning opportunities designed to promote a growth mindset include allowing students to make corrections on their work, supporting students where they are in their learning, having students help each other and the messaging to students in the classroom when class discussions are taking place (Design Principle 3).

The Student Impression Survey has multiple Likert questions associated with this question, including

- The warm-up activities at the start of class are helpful to my learning
- The study skill videos I watched in discussion helped me with my study skills
- I am better able to self-assess if I have fully understood a topic or if I need more instruction on the topic
- I feel there is an emphasis placed on learning from mistakes in this course

- The TED talk videos we watched in class helped me understand how to learn math

I coded each of the responses 4 = "Strongly Agree", 3 = "Agree, 2 = "Disagree", and 1 = "Strongly Disagree". Students had a mixed reaction to the learning opportunities designed to promote a growth mindset. The students agreed or strongly agreed that they are better able to self-assess their abilities and there is an emphasis placed on learning from mistakes in the course. About half the students perceived that the TED talks helped, and half did not. Most students perceived the warm-up activities were beneficial to their learning. Most of the students perceived that the study skills videos they watched at the beginning of the semester were beneficial. See Table 23 for a summary of the responses.

Table 23

Perception of Promoting a Growth Mindset

	Strongly Agree/Agree	Strongly Disagree/Disagree
The warm-up activities at the start of class are helpful to my learning.	40	16
The study skill videos I watched in discussion helped me with my study skills.	32	24
I am better able to self-assess if I have fully understood a topic of if I need more instruction on the topic.	40	6
I feel there is an emphasis placed on learning from mistakes in this course.	40	6
The TED talk videos we watched in class helped me understand how to learn math.	28	18

In the focus group, one student commented on the retakes and correcting mistakes by saying, "for people who are in this math class we like chances." We were able to talk a little more in the focus groups about the class compared to answering specific short answer questions in a survey. On student commented in the focus group that, "if it was classwork, [the teacher] would still have us put it up on the board, which was helpful, and made us feel – or made me feel more comfortable volunteering." Another student mentioned, "I actually – I think, after doing this class, I actually think I like math, now. I used to not like math, actually – well, yeah, actually I really hated math, but I'm starting to lighten up to math." This was said by a couple of the students in the focus groups.

The results from this question are consistent with the findings from previous questions. Students find the aspects of the course beneficial, but the most beneficial aspects of the course are the ones that they perceive directly relate to their mathematical learning. The Likert responses from this question do indicate that students did see a benefit to study skills instruction and TED talks, but as we saw before these were ranked as least important to their learning. Therefore, while beneficial, students may perceive these aspects as less beneficial than other aspects of the course. I will go on to discuss the last research question and then talk about some overall results from the findings.

5.2.3.4 What are students' perceptions of learning opportunities to promote a conceptual understanding of integers and fractions and their operations? The course design focuses on promoting a conceptual understanding of basic mathematics and beginning algebra (Design Principle 2). The foundation of these topics is a conceptual understanding of integers and fractions and their operations. Therefore, much of this course is devoted to these concepts. The learning opportunities afforded to the students related to these topics are daily work and some conceptual understanding activities (see Appendix H).

Students were asked Likert questions in the Student Impression Survey (see Appendix B) that related to this question, including

- I have learned new ways of looking at math problems because of discussing the warm-up activities
- The application problems help me see the connection between topics
- The order of the material has helped me understand topics more compared to my previous math classes
- The in-class activities have helped me gain a better understanding of mathematics
- Learning different methods of solving problems has helped me understand the material better
- Using multiple representations for topics helps with my understanding

I coded each of the responses 4 = "Strongly Agree", 3 = "Agree, 2 = "Disagree", and 1 = "Strongly Disagree". Student perception to the learning opportunities to promote a conceptual understanding showed that the overwhelming majority either strongly agreed or agreed that that the learning opportunities helped them gain a conceptual understanding. See Table 24 for a summary of the responses.

Table 24

Perceptions to Promote a Conceptual Understanding

	Strongly Agree/Agree	Strongly Disagree/Disagree
I have learned new ways of looking at math problems because of discussing the warm-up activities	41	15
The application problems help me see the connection between topics.	44	12

The order of the material has helped me understand topics more compared to my previous math classes.	49	7
The in-class activities have helped me gain a better understanding of mathematics.	48	8
Learning different methods of solving problems has helped me understand the material better.	49	7
Using multiple representations for topics helps with my understanding	47	9

In the focus groups we were able to narrow down some of the questions to students' perceptions of the different teaching techniques. When students were asked if they think they could teach a friend about fractions and integers now, they said, yes. The students explained that they might have to review a little, but they would be able to do it.

Another student commented on how the material was discussed. They mentioned that "people in [this class] had to go through a lot more – not, like slower, but it was more like in-depth, to the point you really can't forget everything." They go on to mention, "I could probably remember more than if I – than I would have remembered, like, in high school." Students seem to know that the work in this course was designed for a deeper understanding than they had received in previous classes. Another student mentioned, "I agree, because it's more like you don't want it to be memorized. You want us to understand the concepts."

The findings suggest that students understand that the design features of the course are put in place to enable them to gain a deeper understanding of the material in the course. It seems that the course design has helped students understand the material and they are better able to recall how to do it in the future. I will go on to discuss some overall findings.

5.3 Summary of Findings

This course has four design principles to promote student success in a developmental mathematics course. Overall the students found the design features that are associated with the design principles to be beneficial to their learning. The design features, including watching videos, study skills instruction and watching TED talks, students did not see a direct benefit to their learning and therefore did not see it as beneficial.

One theme that came out throughout the analysis was the amount of work required for this course. The course is a six-credit course towards financial aid, but zero credits towards graduation. There were a number of students that commented in the short answer question about what one thing is you would change about the course, that they would like to change the workload. They felt the course has “a lot of busy work” and “everyone isn’t a full-time student”. The students commented that the amount of work was a lot to handle and caused stress. Students in this class are expected to spend about two hours outside of class for every credit hour of the course. Thus, it is expected that students spend about 12 hours outside of class on work for this course. The course itself has topics that range from basic math through beginning algebra. This is a lot of material to discuss in 15 short weeks. The comments are understandable, and the instructor should regularly address the reason for the amount of work entailed in the course.

Even though the course is a lot of work, the students step up to the challenge and succeed. They gain a conceptual understanding of the material and understand why the course is designed the way it is. Although there are design features that students don’t perceive as important, the perception might change if the messaging is different from the instructors.

The students in the study passed with an 80% pass rate for the class, and the pass rate did not differ for students' ACT Math score or race. Additionally, the students that enrolled in a math class the next semester had a 74% pass rate. Last, when assessed on conceptual understanding of fractions and integers, the students increased their proficiency on all learning outcomes. This is considerably different compared to students in the traditional sequence of one developmental math course a semester.

Chapter 6 Discussion

6.1 Introduction

This study was a design experiment used to determine if using a holistic approach to designing a developmental mathematics class that addressed many aspects of students referred to multiple levels of developmental mathematics enabled students to succeed in learning mathematics. All of the students in this course have had on average three years of high school mathematics, but their ACT Math exam results and their WI Placement exam results show that the math they have been exposed to has not sunk in. Therefore, the course design has to be more than simply the math topic instruction. Thus, the design principles include (1) creating an equitable environment where students are comfortable participating in mathematical discourse, (2) organizing learning outcomes to promote conceptual understanding and procedural fluency among mathematical objects, (3) providing learning opportunities about different affective characteristics to promote a positive attitude change in a students' mathematical mindset, and (4) creating an assessment system that explicitly values learning. These design principles provide one approach to address the history that developmental mathematics students have as they are starting this course. By providing a holistic approach to the design of the class, students are enabled to not only learn math, but also learn how to learn math.

The first design feature – creating an equitable environment – was implemented because students that enter this class are mostly first-year students and usually first-generation college students. This indicates that many of these students are students of color (Crisp & Delgado, 2014) and have a lower SES compared to the average first-year student (Bailey, 2009).

By using the principles of culturally responsive teaching (Hammond, 2015), students are able to feel safe and comfortable in the classroom environment. This design feature is one of the least visible to the students, but I would argue the most beneficial to their learning. As they begin to feel safe and comfortable, they begin to engage in mathematical discourse inside and outside the classroom. In addition, the equitable environment provides an atmosphere where students are afforded the opportunity to learn in a safe place. In the focus groups the students consistently talked about the sense of community they felt with their classmates. They talked about how they had exchanged phone numbers with others in the class and talked about math outside of class time. Additionally, they discussed how the classroom environment allowed them to learn from each other. The students in the focus groups felt this classroom environment was a benefit to their learning.

The second design feature – promoting a conceptual understanding and procedural fluency among mathematical objects – was implemented to enable students to see the connections between the mathematical objects they have already had exposure to. Students referred to developmental mathematics have a fragmented procedural understanding of mathematics (Stigler et al., 2009). However, these students do have some exposure to mathematics through at least Algebra I and many times Algebra II or higher. This indicates that students have learned some of the mathematics to this point, and this needs to be taken into account when designing the curriculum for the course (Zull, 2002). The work in this course was designed to promote a conceptual understanding of the topics in the course. This study measured fractions, integers, and their operations. The students show statistically significant gains on all learning outcomes that were measured. More importantly, many of the learning outcomes increased to proficient. Since these learning outcomes are a basis of the mathematics they will be learning in the future, this sets the students up for future success.

The third design feature – providing learning activities to promote study skills and a growth mindset – was implemented to provide students with learning opportunities on how to learn math. Acee, et. al. found that developmental mathematics students do not see their math ability as the factor that is impeding their success in their math course. The authors found that self-regulation, study methods, motivation, time management, and stress/anxiety were some of the factors preventing these students from being successful. Thus, the design of this course interweaves learning opportunities about these topics with the mathematical concepts of the course. These learning opportunities afford students the ability to learn how to learn math and practice what they are learning while in the class. The goal is that students will see their success in the class and understand the behaviors that helped them with that success. Then as they progress in their degree path, they will continue to use the tools they learned in this class. The students did not see a direct benefit to the study skills instruction, as they believed they should only be learning math content throughout the course. Conversely, the students mentioned the visible effects of growth mindset instruction like learning from their mistakes, and the instructor's belief that students had the ability to learn the material. In the focus group the students pointed out that the way the class was taught benefited their learning. In fact, one student mentioned the teacher "made me like math."

The fourth design feature – an assessment system that explicitly values learning – was implemented to provide students with the external motivation they may need to complete the course successfully. All activities that will enable students to be successful in the course are assigned grade points. Therefore, there is not one activity, project, or summative exam that will make a student succeed in or fail the course. All activities are important, thus they are all assigned a weight in the course. Most of the activities are approximately an equal weight, this weighting shows students that all aspects of the course are just as important as the other aspects of the course. The learning activities done daily have equal importance to the summative assessments throughout the course. All daily learning activities and most summative assessments are allowed revisions, showing students the importance of learning from their mistakes. All of the grading policies in the course place an emphasis on learning, while still holding the students accountable for their own learning. The design of the course enables students to take control of their learning path and succeed as they move forward. Since all items in the course provide grade points, students see the need to do all the assignments. Because of this, the students are required to do the amount of work required to be successful in the course. The expectation is that students will dedicate about twelve hours of work a week outside of class time since this is a six-credit course. Students feel that the workload for this course quite a bit, but as they start doing the necessary work, they see the benefit of their learning.

Each of these design principles is implemented to meet developmental mathematics students where they are at when they enter the course. They afford students the opportunity to gain a conceptual understanding of basic math and beginning algebra while helping them learn how to learn math. Additionally, each design feature is implemented to maintain high expectations for each student and hold them accountable for their own learning. Throughout this chapter, I will discuss what the findings suggest and the further research that is needed.

6.2 Design Principles compared to Course Structures

In the developmental mathematics community, there has been a discussion of the benefits of using a co-requisite model for developmental mathematics. Thus, much of the research community has been looking exclusively at the course structure of the co-requisite model but has stopped short of looking at design principles and what is going into the course. The co-requisite model has shown promise for developmental mathematics students that require only one level of developmental mathematics. This same model did show marginal gains for students that require multiple levels of developmental mathematics, but the majority of students in this category are still struggling to finish their credit-bearing mathematics courses.

The overall structure of this course is similar to a co-requisite model, in that it combines two courses into one. What sets this course apart are the design principles of the course that address many of the attributes that students requiring multiple levels of developmental mathematics. Therefore, providing a holistic approach to the needs of the students referred to the course. This holistic approach supports students that are referred to multiple levels of developmental mathematics. The approach provides support for many needs of each student. As students progress through the course and subsequent courses they find the success they need to progress. Students in the study that continued in credit bearing mathematics passed their first credit bearing class 71% of the time.

When designing a course for students in this category, all aspects of the students need to be included in the design. By addressing the classroom environment, students are able to learn mathematics in a safe and comfortable environment. Using learning activities that promote a conceptual understanding, students begin to understand the procedures they have be exposed to throughout their mathematics history. Incorporating study skills instruction and growth mindset activities enables students to learn how to learn math, thus preparing them for this course and any subsequent courses. Last, providing the grading structure that values all aspects of this learning motivates students to do the activities they need to do to be successful in the course. These design principles can be incorporated into different course structures. When the value of the course is placed on student learning, students begin to see the same value and see success.

Students that are referred to multiple levels of developmental mathematics have a fragmented understanding of basic mathematics. These students need to develop a solid foundation of these concepts before they proceed into their first credit bearing mathematics course. Corequisite courses usually provide support for one level of developmental mathematics – for instance the credit bearing course is intermediate algebra, then the support course would be beginning algebra. This leaves many concepts unavailable to students that need multiple levels of developmental mathematics. Therefore, these students may not be learning the foundation they need to succeed as they progress in their degree path. It is a disservice to students that are referred to multiple levels of developmental mathematics if they are put in a course that assumes they have proficiency in topics they do not have proficiency and usually there is no support infrastructure to help these students if such gaps are uncovered. In fact, providing support to struggling students and engaging students in conceptual understanding activities have shown to be effective methods of instruction for calculus students (Bressoud & Rasmussen, 2015; Mao, White, & Sadler, 2017) If the topics are not accessible to the students, even with support, they will struggle and give up. This means they will not pass the class they were not prepared to take and face all the repercussions of this outcome.

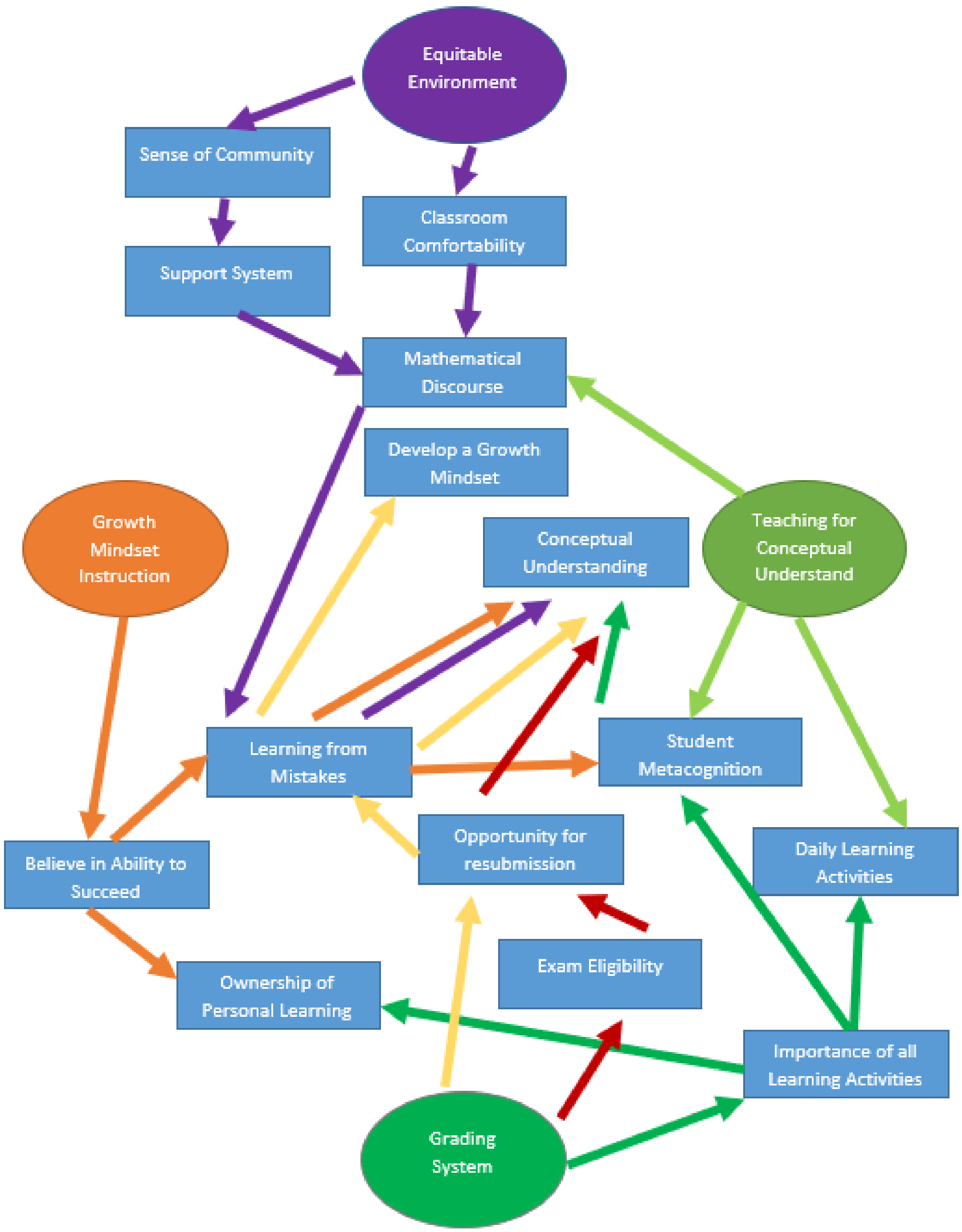
It is important to meet students where they are at and address the gaps and misconceptions, while setting reasonable expectations. Students will meet the expectations that are set if they are reasonable and support is given when students are struggling. Thus, if expectations are set reasonably high students will see the value and meet the expectations that are set. The expectations should be transparent for student learning, thereby allowing students to see the value in the work they are doing. Thus, a developmental mathematics course can be designed with the rigor of a credit bearing mathematics course and still provide the support students need to be successful.

6.3 Holistic Design

The design principles of the course fit together to provide the students in the course the greatest opportunity for success (see Figure 20). Each of the principles are research-based individually, but this study focuses on how they all work together. Below is an overview of how many of the principles of the course fit together. It is challenging to tease out any one feature of the course as being most important or least important. For the remainder of this section, I will provide some evidence on how the different design principles along with the design features used to implement the principles work together to provide a foundation for student learning now and in subsequent courses.

Figure 20

Design Principles Interconnectedness



The first example of this interdependence when the students talk about the community they have built in the course (purple path). The community that has been built started with the equitable environment created by the instructors. The classroom community leads to a comfortability within the classroom, and outside the classroom. Students reach out to each other when they are struggling with material in the course. The students feel comfortable talking with each other about the mathematics. Because of this comfortability, in the class mathematical discourse occurs. The discourse is facilitated by the instructor and enables students to gain a conceptual understanding of the learning objectives in the course. As students gain comfortability talking about mathematics, they are learning the mathematics. In fact, during the focus group one student mentioned how they were learning mathematics by teaching it to their fellow classmates. This mathematics discussion facilitates the students deeper thinking of mathematics.

The grading system provides a place for students to understand the importance of their own learning and develop the habits to be responsible for their own learning (dark green path). The course itself is designed with student learning at the forefront. Thus, all course components are implemented with student learning in mind. Therefore, all course components have grade points assigned to the activity. This is a transparent way for students to understand the importance of that activities in the course, along with the reasons why they are doing the activities. Because there are grade points assigned to each activity, students must take responsibility for doing each of the activities. When students are struggling in the course, they will be able to determine the types of activities they need to perform to succeed in the course because of the transparency in the grading system. In addition, since the course components are designed with student learning at the forefront, students are afforded the opportunity to gain a conceptual understanding of the learning outcomes of the course. The external motivation of grade points being assigned to all course activities helps students develop an internal motivation of doing the work because they see their success as they do each of the activities.

This course places an emphasis on learning from mistakes (orange path). Student work is shown on the board daily, so that students can see other students' responses and learn what mistakes can be made and what to do to fix them. In a lecture-based class, many times the instructor has well-written notes they are working from or has done the work so many times they rarely make mistakes. Students may incorrectly perceive from these lectures that in order to be good at mathematics they must do the problems perfectly every time. When students see the mistakes, they begin to understand that all people make mistakes when they are learning the material, this includes the students that may not have struggled at the beginning. Additionally, they begin to realize that all students struggle with different learning outcomes and have success with others. These endeavors afford students the opportunity to develop a growth mindset in mathematics, the students begin to understand that the topics in the course are accessible to them as long as they keep trying. They begin to understand they have the ability to succeed at mathematics when they see they are gaining a conceptual understanding of the learning outcomes.

Another way students learn from mistakes is when they resubmit their homework and summative assessments (yellow path). While students are encouraged to correct any mistakes, they have during class for the daily work, it does not always happen. Therefore, students will need to make corrections on any mistakes that are made. By correcting mistakes on daily work and summative assessments students are able to focus on their own learning. This helps them gain a conceptual understanding and the knowledge that just because they had a mistake it does not mean they will never understand the material. In this course students are afforded the opportunity to show understanding throughout the course.

Students in this course are required to qualify to take exams (red path). By doing the work necessary to have success on summative exams, students perform better on those exams. To qualify for exams, students need to complete all the workbook assignments that are being assessed on the exam, and all of the online homework assignments being assessed on the exam. As students work though the material in a low-stakes environment, they are able to make mistakes and learn where they have misconceptions. Furthermore, as was discussed previously, students gain a better understanding of the course learning objectives as they work through their mistakes. This is all done in a low-stakes comfortable environment where students are afforded the opportunity to learn without judgement. Then, because students are required qualify for exams, when they get to the exam, they understand how to do the questions on the exam. This leads to students being successful on summative exams.

All of the design principles work together to provide a holistic approach to student learning. As evidence from above, each of the principles intertwine to provide students the support they need to be successful in the course. In addition, each of the principles meet students where they are at when they walk in the door each day and gives them a place to move forward in their learning. Last, the combination of the design principles make students responsible for their own learning, with the instructor as the facilitator to this learning. Students are held to high expectations, and when they meet these expectations, they have success.

6.4 Student Success

Students experience success in this course. The overall pass rate for the course was 79.5%, with students having an ACT Math score of 16 or less having a pass rate of 81%. Additionally, the results showed that there was no difference in pass rates based on race of the students in this study. Furthermore, an additional 8% of the students showed enough progress of the learning outcomes to progress to a co-requisite course or a quantitative literacy course. These are promising results because the students with ACT Math score of 16 or less struggle to pass courses that are designed to only have one level of developmental mathematics combined with a credit-bearing mathematics class. Thus, students in this course are given the opportunity to have success in mathematics.

All the students in the study had a 58% pass rate in their math class the next semester. This number includes 8 students that did not enroll in any classes and 8 students that did not enroll in math class the next semester. Of the students that enrolled in a class the next semester, there is a 79% pass rate. In addition, three students took their algebra course over the winter break and then took Business Calculus in the spring. All three students passed their Business Calculus course. This is the same or larger than other studies that involved enrolling all students into a co-requisite math class no matter their ACT Math score. This shows promising results that students at all ACT Math levels can have success in their gateway math course and proceed to degree completion, because it provides the support the students need when they enter the door. These pass rates are considerably different than the pass rates of students in a traditional developmental sequence when they are referred to multiple levels of developmental mathematics. Fong, et. al., (2014), found that only 12% of the students referred to pre-algebra passed intermediate algebra.

Students also had statistically significant gains in all learning outcomes that were in the study. Students showed an understanding of fractions, integers, and their operations. This includes many of the learning outcomes moving from not proficient to proficient levels. This indicates students are gaining the conceptual understanding necessary for success in their mathematics path.

The holistic design of the course provides the support students need to have success in mathematics. The environment provides the comfort students need to be ready to learn, the learning activities provide students the lessons they need to gain a conceptual understanding of the material, the growth mindset instruction helps them understand they can learn the material, and the grading system provides the extrinsic motivation to help them do the work for the course. All of these design principles together provide the support for students to realize they can do math. This helps them progress on their degree path and moves them forward toward graduation.

6.5 Instructors

The workload for this course for instructors is similar to the workload it is for the students. Since students are required to turn in work on an almost daily basis, the instructors are required to grade on an almost daily basis. The feedback on the graded work is not as extensive as one would expect. Since students are required to make corrections on their own work, lengthy explanations of the mistakes are counterproductive. The feedback should indicate which problems require corrections and possibly a quick hint where the student should start with the correction. Therefore, the daily homework is manageable for grading.

Another concern for workload of the course is requiring students to submit corrections for all their written work. It seems this would be a tremendous amount of work, but it is not. At the beginning of the semester the instructor must set the expectations high for the written work. This is the time when students are getting to know the instructor and they are trying to understand the instructor’s expectations. Students will meet the expectations that are set for them. Thus, if the expectations are set high and the instructor requires work to be written correctly and accurately, students will meet the expectations. Then as the semester progresses the students know what is expected and the workbook sections submitted are done to the expectations that are set. Once the first few workbooks have had a feedback loop, most of the students turn in their workbooks complete and correct with very few if any corrections. The upside to this feedback loop is that instructors are provided insight into where students are struggling immediately, and if the majority of students are struggling with some learning outcomes the instructor is able to facilitate a discussion in class the following day.

The last concern usually raised is the exam qualifications and retake requirements. First, the two weeks leading up to an exam the instructor needs to be in communication with the students if they are missing work, they will need to take the exam. This way students will know the expectations leading up to the exam. As the exam approaches, there will be one or two students that are not eligible to take the exam. These are the students that typically are not participating in class or have missed quite a few classes because of outside obligations on their time. If a student does not qualify to take the exam the first when the exam is originally administered, they are given an extra week to qualify to take the exam. These students are allowed to take the exam when students who need to take a retake are taking the exam. As students qualify to take exams, they begin to see success on their summative assessments. This success has never been realized for some students in mathematics. As the instructor points out that the reason for the success is that they have been doing the work, students are more motivated to do the work.

The design features in the course can lead to a large workload for instructors. But, the benefits of seeing student success far out-weighs the workload. Throughout the semester, the workload equalizes. Thus, at the beginning of the semester the workload is higher as expectations are set for students. Then, as the semester progresses the workload decreases as students meet the expectations. By the end of the semester students are remarking on how much they have learned. This course is extremely rewarding to teach, despite the work involved.

6.6 Recommendations

The design of this course has many parts that are essential to student success. While there are four separate design principles, the principles are intended to work together to provide students with the opportunity for success in this class and any other math classes they want to take. Because there are so many parts, there are some recommendations for students, instructors, and administration.

6.6.1 Student Recommendations

Students value what they feel is beneficial to their learning of the course. Since this course is a mathematics course, students feel that the only topics that should be mathematics topics. The research shows that students that are referred to developmental mathematics also have gaps in their knowledge about how to learn mathematics. Thus, it is imperative that study skills instruction is included in the course. As the study skills instruction is included in the course, the assignments need to be transparent and meaningful. Students should be assessed on the instruction they are receiving. This assessment should include formative assessment by assessing their daily notetaking, determining if students are following through on their workload calendar, and asking students how effective their study habits are becoming. The assessment of the study skills should also include summative assessment on their midterm exams. When items are on the midterm exams students understand the importance of the work they are doing. The study skills assignments must be transparent to students in order for them to see the benefit to their learning.

This course is quite a bit of work for students. At the beginning of the semester the amount of work the course entails must be talked about for students. Many students do not understand why a class that has zero graduation credits requires so much work. The students need to be reminded that the course is comprised of about five years of mathematics from their K-12 experience. The topics are arranged differently, but it is an immense amount of work that needs to be completed. The expectation of completing all the work is for the benefit for the student, and this needs to be reinforced every time a student begins to complain about the workload for the course. The instructors can emphasize with the students, but the expectations must be maintained. Students have the ability to do the work, and when they do, they have success in mathematics.

Students overwhelming did not feel the growth mindset videos benefited their learning. If an instructor is implementing watching the videos they must do so with transparency and have a learning activity associated with it. For instance, the video could discuss mastery-based learning and the learning activity may be a free write about how they have previously been taught mathematics. After the free write, students could get together in groups and talk about how they might benefit from a mastery-based learning approach. This may be an uncomfortable activity for some students as they are expecting to be only learning mathematics in a math course.

This course is different than many math courses that students have taken previously. The instructors must set expectations for students the first day of class and hold those expectations throughout the course. There are aspects of the course that students may find uncomfortable, but with instructor support students will find success in math that they may never have had previously.

6.6.2 Instructor Recommendations

The instructors for this course teach not only the mathematics, but also how to learn mathematics. This may be uncomfortable for higher-ed instructors. Many instructors of developmental mathematics have advanced degrees in mathematics or a math related field, but many do not have much if any pedagogical training. In particular, instructors may have not experienced an active learning classroom or have had training in active learning classrooms. The current recommendation by the MAA is for instructors to teach in an active learning classroom that fosters student engagement (MAA, 2018). Instructors need to have the support they need to develop the skills for facilitating an active learning classroom.

It is possible they have never struggled learning mathematics, or the math they struggled with is far beyond the mathematics being taught in a developmental mathematics class. Thus, it may be difficult for instructors to understand the difficulties students have learning the material. If possible, instructors should find resources that show how to break down the topics in the course. As the instructors learn different ways of teaching the topics, they will be better able to identify the different gaps and fragmented knowledge with which students come into the course.

Instructors should understand the amount of work that focusing on student learning requires. If learning is the ultimate goal, the instructor needs to make the learning accessible to the students where they are at – within reason. It is possible that students have outside obligations that prevent them from completing work until the weekend and then the student turns in all the work on Monday. Therefore, as the work comes in, it is best to grade it and not wait too long. If the grading is done on a daily basis, the workload for the course itself is manageable. It is beneficial to the instructors to grade all the formative work as the instructors are able to see the student progression throughout the semester. The growth can be amazing from some students.

This course meets four days a week, and also requires students to conference with the instructor at least once in the semester. The course design creates an environment where students feel comfortable to learn. This also means it can create an environment where students see the instructor as someone, they are comfortable talking about non-math related topics. Since many of these students are freshmen and first-generation students this point of contact may be what students need to find different campus resources. This may be an uncomfortable position for instructors to have a close relationship with students. If it is uncomfortable, instructors should have a point of contact to refer students in case they need resources.

6.6.3 Administration Recommendations

This course takes resources for instructors to be successful. The most important resource is a central person or team to coordinate the course. The course coordinator must understand the design principles of the course and how they work in conjunction with each other. As the course coordinator works with the instructors of the course, they are better able to address concerns that arise. The course coordinator should be transparent with the design principles of the course and let instructors know how they all work together to focus on student success.

The course coordinator should be given enough support to attend professional development throughout the year. This professional development can include various conferences and the ability to purchase any books or resources that may be beneficial. The support should also include the time required to participate in the conferences and read the books. After participating in professional development, the course coordinator should be held accountable and submit a report to their chair about what they have learned from the professional development. The course coordinator should also relay the information they have learned from the professional development to the instructors of the course.

The course should be staffed with the most experienced active learning instructors available and the instructors should also be given support for professional development. This can include various conferences, school sponsored professional development, and reading books and research. Many of the instructors of developmental mathematics are adjunct instructors or teaching assistants. By providing support for professional development, they will understand the research underlying the design of the course.

This course has many facets to be successful. As instructors and administration work together to give students the tools they need, the students will learn to be successful and progress on their degree path. These students are given an opportunity to begin a college career. This is one way for students to realize the success they need to have to progress on their degree path.

6.7 Future Research

The design approach to this course is one way to meet students where they are at when they enter college needing multiple levels of developmental mathematics. The co-requisite model is a different approach to a similar level of instruction. Currently, many states and college systems are encouraging schools convert their developmental mathematics courses to corequisite models. This model is promising for students in need of one level of developmental mathematics but may leave students needing multiple levels still struggling. By meeting students where they are at, it affords them the opportunity to be successful. There are many different ways that this could be done. Instead of researching specific models of developmental mathematics, we should be researching the aspects of the implementation that are helping students succeed.

Additionally, it is a disservice to students if they are referred to a course that they are not prepared to take. If a corequisite course is only teaching the topics of one developmental class, students that are referred to multiple levels of developmental mathematics are not prepared to be successful.

The design of this class places an emphasis on learning from mistakes and revising daily work and summative assessments to learn from these mistakes. This includes allowing students to retake summative assessments if they did not show proficiency in the material. Students in the class saw the benefit, and the proficiency level increase seems to indicate this is an important part of learning to gain a conceptual understanding. Future research should include the effects of having these types of policies in place for students.

Another area that should be explored is why are students withdrawing and failing the class. Of the students in the study, there were four students that withdrew and nine students that earned an F grade. Of the students that earned the F grade, seven stopped coming to class before the fourteenth week of class. It would be interesting to find out why students stop coming to class. Are students not coming to class because of outside of school obligation, other class obligations, or the course workload? By finding out what is impeding students from coming to class, we might be able to find the support these students need to continue, and progress to degree completion.

Last, another area of future research is what types of professional development are needed for adjunct instructors to be successful teaching this type of course. Since many of these courses are taught by adjunct instructors or teaching assistants, the types of professional development and support that are needed is of great interest. In this course, I provided a two-day course meeting at the beginning of the semester and meetings every two weeks throughout the semester. The two-day course meeting is intended to provide instructors with a foundation of the design principles and instruction on how to conduct the course. The instructor meetings are designed to talk about any issues instructors have had, tips for teaching, and growth mindset instruction for the instructors. Future research should include what other types of professional development is needed for instructors to feel confident teaching this type of course.

6.8 Conclusion

This course provides a holistic approach to designing a course for students in need of multiple levels of developmental mathematics. The design principles in the course are intended to work in collaboration with each other to provide the support students need to obtain success in mathematics. The course meets students where they are at when they enter the door, provides reasonable expectations for student learning, and supports students to facilitate them meeting the expectations. Students obtain the foundation they need for success in future math courses. This puts them on the path toward degree completion.

REFERENCES

Acee, T. W., Barry, W. J., Flaggs, D. A., Holschuh, J. P., Daniels, S., & Schrauth, M. (2017). Student-Perceived Interferences to College and Mathematics Success. *Journal for Developmenal Education*, *40*(2), 2–9.

ACT College and Career Readiness Standards. (2017). Retrieved from http://www.act.org/content/dam/act/unsecured/documents/CCRS-MathStandards.pdf

AMATYC. (2018). *AMATYC Impact: Improving Mathematical Prowess and College Teaching*. (M. B. Orrange & N. J. Sattler, Eds.).

Anderson, T., & Shattuck, J. (2012). Design-Based Research : A Decade of Progress in Educational Research? *Educational Researcher*, *41*(1), 16–25.

Anthony, G. (1996). Active Learning in a Constructivist Framework. *Educational Studies in Mathematics*, *31*(4), 349–369.

Austin, K. (2018). Letter to the Editors: Teaching fractions. *Mathematics in School*, *10*(5), 4–5.

Bailey, T. (2009). Rethinking developmental education in community college, 1–4.

Bailey, T., Jaggars, S., & Scott-Clayton, J. (2013). Characterizing the Effectiveness of Developmental Education: A Response to Recent Criticism. *Community College Research Center*, 1–49. Retrieved from http://www.delta.edu/files/DevEd/Research/response-to-goudas-and-boylan.pdf

Bailey, T., & Jeong, D. W. (2010). Student progression through developmental sequences in community colleges. *Community College Research Center*, 1–6.

Bailey, T., Jeong, D. W., & Cho, S. W. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review*, *29*(2), 255–270. https://doi.org/10.1016/j.econedurev.2009.09.002

Berger, R. (2013). Now I see it, now I don't: researcher's position and reflexivity in qualitative research. *Qualitative Research*, 1–16. https://doi.org/10.1177/1468794112468475

Bickerstaff, S., & Fay, M. P. (2016). Modularization in Developmental Mathematics in Two States : Implementation and Early Outcomes. Retrieved from https://pdfs.semanticscholar.org/7690/a7f654c1b14b02f8d0828188e5d84a93d966.pdf

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through createive math, inspiring messages and innovative teaching*. John Wiley & Sons, Inc.

Bofferding, L. (2009). Addition and Subtraction With Negatives: Acknowledging the Multiple Meanings of the Minus Sign.

Bolyard, J., & Moyer-Packenham, P. (2012). Making sense of integer arithmetic: The effect of using virtual manipulatives on students' representational fluency. *Journal of Computers in Mathematics and Science Teaching*, *31*(2), 93–113. Retrieved from http://www.editlib.org/p/39192/paper_39192.pdf

Bonham, B. S., & Boylan, H. R. (2011). Developmental Mathematics: Challenges, Promising Practices, and Recent Initiatives. *Journal of Developmental Education*, *34*(3), 2–10.

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education*, 119–15.

Boylan, H. R., & Bonham, B. S. (2007). 30 Years of Developmental Education: A Retrospective. *Journal of Developmental Education*, *30*(3), 2–4. Retrieved from http://libproxy.txstate.edu/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=ehh&AN=25122568&site=ehost-live

Bracco, K. R., Austin, K., Bugler, D., & Finkelstein, N. (2015). Reforming Developmental Education to Better Support Students' Postsecondary Success in the Common Core Era. Core to College Evaluation. Policy Brief. *WestEd*, (June 2015).

Bressoud, D., & Rasmussen, C. (2015). Seven Characteristics of Successful Calculus Programs. *Notices of the AMS*, 144–146.

Brock, T. (2010). Young adults and higher education: barriers and breakthrouhgs to success. *The Future of Children*, *20*(1), 109–132. https://doi.org/10.1353/foc.0.0040

Brothen, T., & Wambach, C. A. (2004). Refocusing developmental education. *Journal of Developmental Education*, *28*, 16–21. https://doi.org/10.2753/RPO1061-0405490603

Burley, H., Butner, B., & Wilson, C. (2009). Predicting Bachelor ' s Degree Attainment for Developmental Math Students, *2000*(2002), 29–40.

Cafarella, B. B. (2016). Acceleration and Compression in Developmental Mathematics: Faculty Viewpoints. *Journal of Developmental Education*, *39*(2), 12–25.

CBMS. (n.d.). CBMS Member Societies. Retrieved February 7, 2019, from https://www.cbmsweb.org/member-societies/

CBMS. (2016). *Active Learning in Post-Secondary Mathematics Education*.

Choy, S. P., Horn, L. J., Nuñez, A., & Chen, X. (2000). Transition to College: What Helps At-Risk Students and Students Whose Parents Did Not Attend College. *New Directions for Institutional Research*, (107), 45–63.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in

Educational Research. *Educational Researcher*, *32*(1), 9–13. https://doi.org/10.3102/0013189X032001009

Cobb, Paul, Zhao, Q., & Dean, C. (2009). Conducting design experiments to support teachers' learning: A reflection from the field. *Journal of the Learning Sciences*, *18*(2), 165–199. https://doi.org/10.1080/10508400902797933

Collective, D.-B. R. (2003). Design-based research : An emerging paradigm for educational inquiry. *Educational Researcher*, *32*(1), 5–8.

Cramer, K. A., Post, T. R., Robert, C., Cramer, K. A., Post, T. R., & Robert, C. (2018). Initial Fraction Learning by Fourth- and Fifth-Grade Students : A Comparison of the Effects of Using Commercial Curricula with the Effects of Using the Rational Number Project Curriculum Linked references are available on JSTOR for this article : Initial. *Journal for Research in Mathematics Education*, *33*(2), 111–144.

Creswell, J. W., & Creswell, J. D. (2018). *Research Design: Qualitative, Quantitative and Mixed Methods Approaches* (Fifth). Thousand Oaks, CA: SAGE.

Crisp, G., & Delgado, C. (2014). The impact of developmental education on community college persistence and vertical transfer. *Community College Review*, *42*(2), 99–117.

Denley, T. (2017). CoRequisite Developmental Mathematics. Retrieved from https://sites.nationalacademies.org/cs/groups/dbassesite/documents/webpage/dbasse_191879.pdf

Dweck, C. S. (2008). *Mindset: The new psychology of success*. Random House Digital, Inc.

Ehlert, D. (2015). Promoting Growth Mindset through Assessment. Retrieved from https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Promoting-Growth-Mindset-through-Assessment/

Fong, K. E., Melguizo, T., & Prather, G. (2015). *Increasing Success Rates in Developmental Math: The Complementary Role of Individual and Institutional Characteristics*. *Research in Higher Education* (Vol. 56). https://doi.org/10.1007/s11162-015-9368-9

Fong, K. E., Melguizo, T., Prather, G., & Bos, J. M. (2014). A Different View of How We Understand Progression Through the Developmental Math Trajectory, 1–6.

Gerdes, H., & Mallinkrodt, B. (1994). Emotional Social and academic adjustment of college students. *Journal of Counseling & Development*, *72*, 281–288.

Golafshani, N. (2013). Teachers' Beliefs and Teaching Mathematics with Manipulatives. *Canadian Journal of Education / Revue Canadienne de l'éDucation*, *36*(3), 137–159. Retrieved from http://0-www.jstor.org.library.ucc.ie/stable/canajeducrevucan.36.3.137

Goudas, A. M. . A. E., & Boylan, H. R. . (2012). Addressing Flawed Research in Developmental Education. *Journal of Developmental Education*, *36*(1), 2–13. Retrieved from http://ezproxy.shsu.edu/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eft&AN=85225577&site=eds-live&scope=site$%5Cbackslash$nhttp://content.ebscohost.com.ezproxy.shsu.edu/ContentServer.asp?T=P&P=AN&K=85225577&S=R&D=eft&EbscoContent=dGJyM

Graduation Requirements WI DPI. (2019). Retrieved from https://dpi.wi.gov/graduation/requirements

Hall, M., & Ponton, M. K. (2005). Mathematics Self-Efficacy of College Freshman. *Journal of Developmental Education*, *28*(3), 26–32.

Hammerman, N., & Goldberg, R. (2003). Strategies for Developmental Mathematics at the College Level. *Mathematics & Computer Education*, *37*(1), 79–95. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=507873156&site=ehost-live

Hammond, Z. (2015). *Culturally Responsive Teaching & The Brain*. Thousand Oaks, CA: SAGE.

Hern, K. (2012). Acceleration Across California: Shorter Pathways in Develpmental English and Math. *Change*, (May/June 2012), 60–69.

Hicks, T., & Heastie, S. (2008). High School to College Transition: A Profile of the Stressors, Physicial and Psychological Health Issues That affect the First-Year on-campus college student. *Journal of Cultural Diversity*, *15*(3), 143–147.

Higbee, J. L., & Thomas, P. V. (1999). Affective and cognitive factors related to mathematics achievenemnt. *Journal of Developmental Education*, *23*(1), 8.

Hodara, M. (2011). Reforming Mathematics Classroom Pedagogy: Evidence-Based Findings and Recommendations for the Developmental Math Classroom. CCRC Working Paper No. 27. Assessment of Evidence Series. *Community College Research Center, Columbia University*, (February). Retrieved from http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED516147%5Cnhttp://www.eric.ed.gov/ERICWebPortal/detail?accno=ED516147

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Educaiton*, *35*(2), 81–116.

Hunt, A. W. A., Nipper, K. L., & Nash, L. E. (2011). Virtual vs. concrete manipulatives in mathematics teacher education: Is one type more effective than the other? *Current Issues in Middle Level Education*, *16*(2), 1–6. Retrieved from http://eric.ed.gov/?id=EJ1092638

Izsák, A., Jacobson, E., Araujo, Z. De, Orrill, C. H., Izsák, A., Jacobson, E., … Orrill, C. H. (2018).

Measuring Mathematical Knowledge for Teaching Fractions With Drawn Quantities. *Journal for Research in Mathematics Education*, *43*(4), 391–427.

Izsák, A., Tillema, E., & Tunc-Pekkan, Z. (2008). Teaching and learning fraction addition on number lines. *Journal for Research in Mathematics Education2*, *39*(1), 33–62.

Jaggars, S. S., Hodara, M., Cho, S., & Xu, D. (2015). Three Accelerated Developmental Education Programs: Features, Student Outcomes, and Implications. *Community College Review*, *43*(1), 3–26. https://doi.org/10.1177/0091552114551752

Jones, S. (2015). The game changers: Strategies to boost college completeion and close attainment gaps. *Change*, (March/April). Retrieved from https://biblio.ugent.be/publication/4207426/file/6806475

Khan, S. (2015). Let's teach for mastery - not test scores. Retrieved from https://www.ted.com/talks/sal_khan_let_s_teach_for_mastery_not_test_scores

Kiewra, K. A. (2002). How classroom teachers can help students learn and teach them how to learn. *Theory into Practice*, *41*(2), 71–80. https://doi.org/10.1207/s15430421tip4102_3

Kilpatrick, J., Swafford, J., & Bradford, F. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Research Council and Natinal Academy Press.

King, J. B., Mcintosh, A., & Bell-Ellwanger, J. (2017). *Developmental Education Challenges and Strategies for Reform*. Retrieved from https://www2.ed.gov/about/offices/list/opepd/education-strategies.pdf

Kosiewicz, H., Ngo, F., & Fong, K. (2016). Alternative models to deliver developmental math: Issues of use and student access. *Community College Review*, *44(3)*, 205–231.

Larsen, J. (2015). Adult students' experiences of a flipped mathematics classroom. *Adults Learning Mathematics: An International Journal*, *10*(1), 50–67.

Linchevski, L., & Williams, J. (1999). Gap in Children ' S Extension of Their Number Concept. *Educational Studies in Mathematics*, 131–147. https://doi.org/10.1023/A:1003726317920

MAA. (2018). *MAA Instructional Practices Guide*. Retrieved from https://www.maa.org/sites/default/files/InstructPracGuide_web.pdf

Mangan, B. K. (2014). Push to Reform Remedial Education Raises Difficult Questions for Colleges, 1–13. Retrieved from http://chronicle.com/article/Push-to-Reform-Remedial/145817/

Mao, Y., White, T., & Sadler, P. M. (2017). The association of precollege use of calculators with student performance in college calculus. *Educational Studies in Mathematics*, 69–83. https://doi.org/10.1007/s10649-016-9714-7

McKay, S. (2015). Using New Research to Improve Student Motivation | Carnegie Foundation for the Advancement of Teaching. Retrieved from https://www.carnegiefoundation.org/blog/using-new-research-to-improve-student-motivation/

Melguizo, T., Bos, J., & Prather, G. (2011). Is Developmental Education Helping Community College Students Persist? A Critical Review of the Literature. *American Behavioral Scientist*, *55*(2), 173–184. https://doi.org/10.1177/0002764210381873

Mireles, S. V, Acee, T. W., & Gerber, L. N. (2014). FOCUS: Sustainable Mathematics Successes. *Journal of Developmental Education*, *38*(1), 26–36. Retrieved from https://ezp.lib.unimelb.edu.au/login?url=https://search.ebscohost.com/login.aspx?direct=true&db=ehh&AN=101143300&site=eds-live&scope=site

Moore, A. J., Gillett, M. R., & Steele, M. D. (2014). Fostering Student Engagement with the Flip. *The Mathematics Teacher*, *107*(6), 420. https://doi.org/10.5951/mathteacher.107.6.0420

Moreno, R., & Mayer, R., E. (1999). Multimedia-supported metaphors for meaning making in mathematics.

Moss, P. A., & Haertel, E. H. (2016). Engaging methodological pluralism. In D. Gitomer & C. Bell (Eds.), *Handbook of research on teaching* (5th ed., pp. 127–247). Washington, DC: AERA.

Moyer, P. S., & Jones, M. G. (2004). Controlling choice: Teachers, students, and manipulatives in mathematics classrooms. *School Science and Mathematics*, *104*(1), 16–31. https://doi.org/10.1111/j.1949-8594.2004.tb17978.x

National Council or Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Natinal Council of Teachers of Mathematics (NCTM).

National Research Council. (2003). *Helping Children Learn Mathematics*. (Jeremy Kilpatrick & J. Swafford, Eds.). Washington, DC: National Academy Press.

Ngo, F., & Kosiewicz, H. (2017). How Extending Time in Developmental Math Impacts Student Persistence and Success: Evidence from a Regression Discontinuity in Community Colleges. *The Review of Higher Education*, *40*(2), 267–306. https://doi.org/10.1353/rhe.2017.0004

Nolting, P. (2014). *Winning at Math* (6th ed.). Academic Success Press.

Nomi, T., & Allensworth, E. (2009). Intervention, evaluation, and policy studies: 'Double-dose' algebra as an alternative strategy to remediation: Effects on students' academic outcomes. *Journal of Research on Educational Effectiveness*, *2*(2), 111–148. https://doi.org/10.1080/19345740802676739

Peled, I., & Carraher, D. W. (2007). Signed Numbers and Algebraic Thinking. *Algebra in the Early Grades*, (January 2008), 303–328.

Pillow, W. S. (2003). Confession, catharsis, or cure? Rethinking the uses of reflexivity as methodological power in qualitative research. *International Journal of Qualitative Studies in Education*, *16*(2), 175–196. https://doi.org/10.1080/0951839032000060635

Probst, B. (2015). The eye regards itself: Benefits and challenges of reflexivity in qualitative social work research. *Social Work Research*, *39*(1), 37–48. https://doi.org/10.1093/swr/svu028

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the Conceptual Structure of Mathematics. *Educational Psychologist*, *47*(3), 189–203. https://doi.org/10.1080/00461520.2012.667065

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, *91*(1), 175–189. https://doi.org/10.1037/0022-0663.91.1.175

Rutschow, E. Z., & Schneider, E. (2011). Unlocking the Gate: What we know about improving developmental education. *MDRC*, (June).

Scott-Clayton, J., Crosta, P. M., & Belfield, C. R. (2014). Improving the Targeting of Treatment. *Educational Evaluation and Policy Analysis*, *36*(3), 371–393. https://doi.org/10.3102/0162373713517935

Sfard, A. (1991). On the Dual Nature of Mathematical Conceptions : Reflections on Processes and Objects as Different Sides of the Same Coin Author (s): Anna Sfard Source : Educational Studies in Mathematics , Vol . 22 , No . 1 (Feb ., 1991), pp . 1-36 Published by : Sp. *Educational Studies in Mathematics*, *22*(1), 1–36.

Sheldon, C. Q., & Durdella, N. R. (2010). Success rates for students taking compressed and regular length developmental courses in the community college. *Community College Journal of Research and Practice*, *34*(1–2), 39–54. https://doi.org/10.1080/10668920903385806

Smith, A. (2015). Florida's remedial law leads to decreasing pass rates in math and English | Inside Higher Ed. Retrieved from https://www.insidehighered.com/news/2015/06/25/floridas-remedial-law-leads-decreasing-pass-rates-math-and-english

Smith III, J. P. (1996). Efficacy and Teaching Mathematics by Telling : A Challenge for Reform. *Journal for Research in Mathematics Education*, *27*(4), 387–402.

Spangler, D. B. (2011). *Strategies for Teaching Fractions: Using error analysis for intervention and assessment*. Thousand Oaks, CA: Corwin.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In *Research design in mathematics and science* *education*.

Stephan, M., & Akyuz, D. (2012). A Proposed Instructional Theory for Integer Addition and Subtraction. *Journal for Research in Mathematics Education*, *43*(4), 428. https://doi.org/10.5951/jresematheduc.43.4.0428

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2009). What Community College Developmental Mathematics Students Understand about Mathematics What Community College Developmental Mathematics Students Understand about Mathematics. *Knowledge Creation Diffusion Utilization*.

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *The MathAMATYC Educator*, *1*(3), 4–18. Retrieved from http://statlit.org/pdf/2009CarnegieFoundation-Developmental-Math-CC-Students-Understand.pdf

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*(2), 213–226. https://doi.org/10.1016/S0742-051X(00)00052-4

Tapia, M. (1996). The attitude toward mathematics instrument. In *Mid-South Education Research Association*.

Taylor, E. (2014). Spending more of the school day in math class: Evidence from a regression discontinuity in middle school. *Journal of Public Economics*, *117*, 162–181.

TBR CoRequisite Study - Update Spring 2016. (2016). Retrieved January 1, 2017, from https://www.tbr.edu/sites/tbr.edu/files/media/2016/04/TBR CoRequisite Study - Update Spring 2016%5B1%5D_1.pdf

Twigg, C. A. (2003). Improving Learning and Reducing Costs. *Educause Review*, *38*(October), 28–38. https://doi.org/10.1016/j.ygeno.2009.08.012

Twigg, C. A. (2011). The math emporium: Higher education's silver bullet. *Change*, *43*(3), 25–34. https://doi.org/10.1080/00091383.2011.569241

Valentine, J. C., Konstantopoulos, S., & Goldrick-Rab, S. (2017). What Happens to Students Placed Into Developmental Education? A Meta-Analysis of Regression Discontinuity Studies. *Review of Educational Research*, *87*(4), 806–833. https://doi.org/10.3102/0034654317709237

Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology*, *21*(4), 555–570. https://doi.org/10.1080/09515080802285552

Wang, T., & Cai, J. (2007). United States teachers' views of effective mathematics teaching and learning. *ZDM Mathematics Education*, *39*(4), 315–327. https://doi.org/10.1007/s11858-007-0033-4

Weimer, M. (2013). *Learner-Centered Teaching: Five Key Changes to Practice* (2nd ed.). San Francisco: John Wiley & Sons, Inc.

Wood, T., Cobb, P., & Yackel, E. (1991). Change in Teaching Mathematics: A Case Study. *American Educational Research Journal1*, *28*(3), 587–616.

Wright, G. L., Wright, R. R., & Lamb, C. E. (2002). Developmental Mathematics Education and Supplemental Instruction: Pondering the Potential. *Journal of Developmental Education*, *26*(1), 30–35.

Zavarella, C. A., & Ignash, J. M. (2009). Instructional Delivery in Developmental Mathematics: Impact on Retention. *Journal of Developmental Education*, *32*(2), 2–13.

Zull, J. E. (2002). *The art of changing the brain: Enriching the practice of teaching by exploring the biology of learning* (1st ed.). Sterling, VA: Stylus Publishing, LLC.

Zull, J. E. (2004). The Art of Changing the Brain. *Educational Leadership*. https://doi.org/10.1017/CBO9781107415324.004

Appendix A – Focus Group Protocol

Hi Everyone,

Thank you for meeting with me and helping me with my research. This focus group should take about one hour. Do you mind if I record the session, so I can transcribe it later? I just want you to know that this focus group will be confidential and the only people that will see the transcription will be me and my dissertation advisor. The purpose of this focus group is to understand your feelings about your math course. Please share whatever you would like. I will not be offended by anything that is said here, and it will in no way affect your final grade in the course.

There are a number of questions that ask whether the activities support your learning – here is what I mean about that:

- Does the activity clarify the topic for you?
- Does the activity help you gain a deeper understanding of the topic?
- Does the activity help you make connections to related topics?
- Do you get a lightbulb moment or a-ha moment when you are doing the activity?

Let's start with some basics:

Hold up on your fingers how well do you feel the course is going 1 being not good and 5 being great.

I'd like to hear from a range of perspectives, would you be willing to explain why you choose that number?

For those of you that showed 1 or 2?

For those of you that showed 4 or 5?

What are the things in the course you feel support your learning?

Why?

What are the things in the course you currently feel don't support your learning?

Why?

There are many class activities throughout the semester. Do you find that these activities support your learning?

What are your feelings on the grading policies?

Do you find that redoing your assignments until they are correct supports your learning?

Why?

Do you find it supports your learning to qualify to take exams?

Why?

How did the study skill instruction in discussion support your learning?

How are mistakes approached in your course? What types of actions does your instructor take when a mistake is made in presenting your material?

Did these actions support your learning?

In-class, was there an open discussion about mathematical concepts? Did these discussions support your learning?

The course content is arranged so that you learn definitions, then operations, and then solving equations and inequalities. Do you think it help you build connections between concepts to have the content arranged this way?

If you were helping a friend in Math 94 next semester, do you think you could explain the material? Why or why not?

Do you have anything else you would like to add or any questions you have for me?

Thank you again for participating in the focus group.

Appendix B: Student Impression Survey

What is your last name?

What is your first name?

Who is your instructor?

Please indicate how strongly you agree or disagree with each of the statements.

	Strongly Agree	Agree	Disagree	Strongly Disagree	RQs	DP
The worksheet corrections helped strengthen my understanding of mathematics.					4b	2, 3
Knowing that I have to qualify to take exams better prepares me for the exams.					4b	3, 4
I feel there is an emphasis placed on learning in this course.					4b	3, 4
Self-correcting my mistakes help me understand my misconceptions.					4b	4
I feel the grading system values my learning					4b	4
I feel that all work I complete in the course has a value.					4b	4
I am glad I have the ability to re-do an exam if I do not perform well the first time.					4b	4
The warm-up activities at the start of class are helpful to my learning.					4c	1, 2
The study skill videos I watched in discussion helped me with my study skills.					4c	2, 3
I am better able to self-assess if I have fully understood a topic or if I need more instruction on the topic.					4c	3
I feel there is an emphasis placed on learning from mistakes in this course.					4c	3
The TED talk videos we watched in class helped me understand how to learn math.					4c	3

I have learned new ways of looking at math problems because of discussing the warm-up activities					4d	2
The application problems help me see the connection between topics.					4d	2
The order of the material has helped me understand topics more compared to my previous math classes.					4d	2
The in-class activities have helped me gain a better understanding of mathematics.					4d	2
Learning different methods of solving problems has helped me understand the material better.					4d	2
Using multiple representations for topics helps with my understanding					4d	2

This course has many course requirements, please rank the following in order from most important to least important for your learning.

Mandatory attendance
Mandatory discussion
In-class activities
Warm-up activities
TED talks
Correcting mistakes
Exam retakes
Online homework
Grading system
Study Skill instructions
Watching videos before class
Order of the topics
Class discussions

Choose one of the top three items you choose as important, why is it important to your learning?

Choose one of the lowest three items you choose as least important, why do you feel this did not help you in your learning?

If you could change one way this class is taught, what would you change to better support future student learning?

If someone you know had to take this class next semester, what would you tell them are the most beneficial parts of this class?

Appendix C – Pre-Test and Final Exam Questions

Name: ______________________________

1. Model the given fraction using a diagram. Divide the number line into equal increments and place your fraction on one of the tick marks. Determine the decimal representation for each fraction.

Fraction	**Diagram Model**	**Number Line Model**	**Decimal Representation**
$\frac{2}{3}$		0 — 3	
$\frac{5}{8}$		0 — 3	
$\frac{7}{6}$		0 — 3	
$\frac{9}{4}$		0 — 3	

2. In each of the following, determine if the fractions are equivalent. Justify your answer by showing work, explaining or drawing a picture.

Equivalent Fractions?	Yes or No	Justify your answer
$\frac{4}{18}$ *and* $\frac{2}{9}$	Yes No	
$\frac{8}{6}$ *and* $\frac{3}{4}$	Yes No	

$\frac{6}{8}$ *and* $\frac{15}{20}$	Yes No	
$-\frac{4}{5}$ *and* $\frac{4}{-5}$	Yes No	
$-\frac{4}{18}$ *and* $\frac{-2}{-9}$	Yes No	

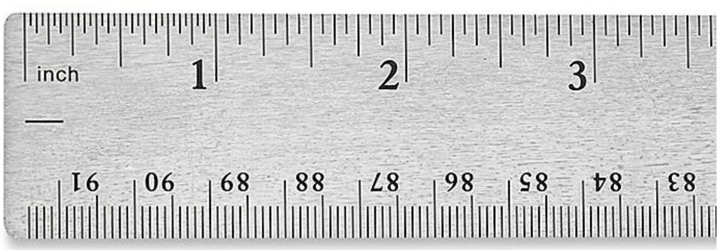
3. Construct a number line. Place the values $-7, 3, 0$, and -4 on the same number line. Each number should land on its own tick mark.

   What increment did you use? ___________

   What is the distance between 3 and -4? _________

   Show the opposite of -4 on the number line.

4. On the ruler below, mark the following: $\frac{3}{4}$ in, $\frac{7}{8}$ in, and $1\frac{5}{8}$ in



   a. What is the distance between $\frac{3}{4}$ in and $1\frac{5}{8}$ in?

   b. Mark the position(s) on the ruler that is a distance of $\frac{1}{2}$ in away from $\frac{7}{8}$ in. Mark it on the ruler.

5. Order the following sets of numbers from least to greatest
   a. $3.25,\ 3.435,\ 3.2,\ \ 3,\ \ 3.258$
      Justify your answer:

   b. $\frac{17}{13}, \frac{3}{13}, 13, \frac{11}{13}, \frac{7}{13}$

      Justify your answer:

   c. $\frac{2}{3}, -\frac{2}{5}, \frac{7}{15}, \frac{5}{3}, -\frac{19}{15}$

      Justify your answer:

   d. $\frac{7}{8}, \frac{3}{4}, \frac{14}{15}, \frac{5}{6}, \frac{20}{21}$

      Justify your answer:

6. Evaluate the following and write your answer as a single fraction:
   a. $\frac{3}{4} + \frac{5}{8}$
   b. $\frac{4}{3} - \frac{7}{3}$
   c. $\frac{2}{3} * \frac{7}{4}$
   d. $\frac{8}{5} \div \frac{10}{4}$
   e. $\frac{7}{10} + \frac{9}{100}$

7. Multiply or divide and write your answer as a single fraction. You must show your work!
   a. $\frac{4x}{3} * \frac{9}{14x}$

   b. $\frac{7y}{9} * 27$

c. $\frac{5}{b} * b$

d. $\frac{9}{7a} \div \frac{7a}{5}$

8. Consider the expressions (a) and (b) below, you should not evaluate them. Reference these two expressions as you respond to parts (c) through (e). Choose the correct statement.

a. $856 + 439$

b. $856 - 439$

c. $-439 + 856$
   i. This is equivalent to (a)
   ii. This is equivalent to (b)
   iii. This is not equivalent to (a) or (b)

d. $439 - (-856)$
   i. This is equivalent to (a)
   ii. This is equivalent to (b)
   iii. This is not equivalent to (a) or (b)

e. $-856 - (-439)$
   i. This is equivalent to (a)
   ii. This is equivalent to (b)
   iii. This is not equivalent to (a) or (b)

9. Predict the sign of the value **N**. You should not perform the calculation.

Equation	Positive or Negative	Equation	Positive or Negative
$-8 * 13 = \boldsymbol{N}$	+ or –	$\boldsymbol{N} - (-10) = 6$	+ or –
$(-7) + \boldsymbol{N} = (-10)$	+ or –	$\boldsymbol{N} * 7 = -28$	+ or –
$\boldsymbol{N} + 14 = 9$	+ or –	$(-5) * (-13) = \boldsymbol{N}$	+ or –
$(-4) * \boldsymbol{N} = 36$	+ or –	$5 - \boldsymbol{N} = -15$	+ or –

10. You decide to have a birthday party for your roommate. You invite 4 people and order 3 extra-large pizzas that are cut into 8 pieces each. There are 6 people at the party. Justify each of your answers.

    If everyone can eat an equal amount, what fraction of one whole pizza does each person get to eat?

    Your friends are going to split the cost with you. What fraction of the bill will each of you pay? (Don't forget your roommate does not need to pay)

    If the total with delivery was $40, how much money does everyone have to give you?

    Not everyone ate their equal share of the pizza. In the end there was 2 pieces left of the pepperoni, 3 pieces left of cheese, and 4 pieces left of the veggie. What fraction of the total amount of pizza ordered is left over?

    Three party crashers join the party and share the leftover pizza. What fraction of one pizza will they be able to eat if they shared it equally?

Appendix D – Attitude Toward Mathematics Index

ATTITUDES TOWARD MATHEMATICS INVENTORY

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Circle the letter that best describes how much you agree or disagree with each statement.

PLEASE USE THESE RESPONSE CODES:
A – Strongly Disagree
B – Disagree
C – Neutral
D – Agree
E – Strongly Agree

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1. Mathematics is a very worthwhile and necessary subject.	A	B	C	D	E
2. I want to develop my mathematical skills.	A	B	C	D	E
3. I get a great deal of satisfaction out of solving a mathematics problem.	A	B	C	D	E
4. Mathematics helps develop the mind and teaches a person to think.	A	B	C	D	E
5. Mathematics is important in everyday life.	A	B	C	D	E
6. Mathematics is one of the most important subjects for people to study.	A	B	C	D	E
7. High school math courses would be very helpful no matter what I decide to study.	A	B	C	D	E
8. I can think of many ways that I use math outside of school.	A	B	C	D	E
9. Mathematics is one of my most dreaded	A	B	C	D	E

subjects.					
10. My mind goes blank and I am unable to think clearly when working with mathematics.	A	B	C	D	E

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
11. Studying mathematics makes me feel nervous.	A	B	C	D	E
12. Mathematics makes me feel uncomfortable.	A	B	C	D	E
13. I am always under a terrible strain in a math class.	A	B	C	D	E
14. When I hear the word mathematics, I have a feeling of dislike.	A	B	C	D	E
15. It makes me nervous to even think about having to do a mathematics problem.	A	B	C	D	E
16. Mathematics does not scare me at all.	A	B	C	D	E
17. I have a lot of self-confidence when it comes to mathematics.	A	B	C	D	E
18. I am able to solve mathematics problems without too much difficulty.	A	B	C	D	E
19. I expect to do fairly well in any math class I take.	A	B	C	D	E
20. I am always confused in my mathematics class.	A	B	C	D	E
21. I feel a sense of insecurity when attempting mathematics.	A	B	C	D	E

22. I learn mathematics easily.	A	B	C	D	E
23. I am confident that I could learn advanced mathematics.	A	B	C	D	E
24. I have usually enjoyed studying mathematics in school.	A	B	C	D	E
25. Mathematics is dull and boring.	A	B	C	D	E

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
26. I like to solve new problems in mathematics.	A	B	C	D	E
27. I would prefer to do an assignment in math to writing an essay.	A	B	C	D	E
28. I would like to avoid using mathematics in college.	A	B	C	D	E
29. I really like mathematics.	A	B	C	D	E
30. I am happier in a math class than in any other class.	A	B	C	D	E
31. Mathematics is a very interesting subject.	A	B	C	D	E
32. I am willing to take more than the required amount of mathematics.	A	B	C	D	E
33. I plan to take as much mathematics as I can during my education.	A	B	C	D	E
34. The challenge of math appeals to me.	A	B	C	D	E

35. I think studying advanced mathematics is useful.	A	B	C	D	E
36. I believe studying math helps me with problem solving in other areas.	A	B	C	D	E
37. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.	A	B	C	D	E
38. I am comfortable answering questions in math class.	A	B	C	D	E
39. A strong math background could help me in my professional life.	A	B	C	D	E
40. I believe I am good at solving math problems.	A	B	C	D	E

DEMOGRAPHIC QUESTIONNAIRE

Please answer the following by filling in the blank with the appropriate information.

Student Identification Number (this number begins with 990 or 991) _______________

Gender (circle one): Male Female

Age __________

Racial/Ethnic Background: (Put an X next to the one that best fits you)

_____ African American
_____ American Indian
_____ Asian American
_____ International student
_____ Latino/a
_____ Multi Ethnic
_____ Other Race/Ethnicity
_____ Southeast Asian American
_____ White

I have taken a Math class at UWM (circle one): Yes No

How many times have you completed Math 094? (Put an X next to the one that best fits you)

____ This is the first time I have taken Math 094.
____ I am repeating Math 094 for the first time (i.e. I took it once before).
____ I have taken Math 094 at least twice before.

From what high school did you graduate?
Name of school: ____
City it is located in: ____

What year did you graduate from high school (or receive GED, if applicable): _________
Year indicated above refers to (circle one): High school graduation GED

Appendix E – Content Ordering

The first chapter of material is definitions of all the mathematical objects that are used throughout the course. These include

- number systems, and the base 10 number system
- the set of complex numbers and its subsets
- fractions and equivalent fractions, and how to model each using pictures and number lines
- the equivalence between fractions, percentages, and decimals
- the coordinate plane and distances, the complex plane and absolute value
- expressions, equations and inequalities – and classifying each of these
- properties of real numbers
- properties of equality and inequality
- exponents, including negative exponents, zero exponents, and rational exponents
- properties of exponents using all the different types of exponents

The sections are specifically designed for students to make connections to the other sections and their prior knowledge. Each section includes applications meant to help students gain a conceptual understanding of each of the topics.

The second chapter is arithmetic operations with the objects that are described in chapter one. The chapter focuses on the similarities of the operation with all the objects. For instance, in the addition section, students are shown how to add integers, fractions, polynomials, measurement units, rational expressions, numbers written in scientific notation, and complex numbers at the same time. This approach allows students to understand the connections between the objects and the operations. The second chapter includes:

- addition
- subtraction
- multiplication
- greatest common factor and least common multiple
- factoring
    - factoring out a greatest common factor
    - factor by grouping
    - factoring using the AC method
    - difference of squares
    - perfect square polynomials
- multiplying rational expressions
- division
- order of operations

The third chapter discusses solving and graphing equations and inequalities. These are solved and graphed together, again for students to understand the similarities between the two. In addition, students are introduced to solving equations and inequalities by determining if a given answer is a solution to the equation or inequality, or system of equations. Next, students are given the opportunity to see the connections with these solutions and non-solutions and how they relate to graphs of lines and systems of equations. The chapter also includes solving linear equations and inequalities, writing linear equations given points or a point and the slope, graphing linear equations and inequalities, solving quadratic equations. Again, applications are included in every section so students can apply what they have learned to real-world situations immediately.

Appendix F – Before Start of the Semester Communication

Email before the class begins

Make sure to attach Syllabus,

course outline,

Hello!
Welcome to Fall 2019 semester. You are receiving this e-mail as you are enrolled in the Math 094 course**. Please respond to this email so I know you received it, if you don't respond I will call to make sure you received the message.**

The primary focus of this course is to emphasize problem solving and is designed to promote seeing connections and patterns between different topics. The course also focuses on preparing you to succeed in your next math classes also by teaching you techniques to help you study better.

This course is an accelerated math course, and we cover two courses of mathematics topics in one semester. The class moves very fast, and therefore you need to make sure to attend every class period.

To get you ready to start the semester, there are two surveys, an initial knowledge check in ALEKS, and a syllabus quiz. You can find the surveys, the quiz and a link to ALEKS in the **Canvas** course page. Canvas can be accessed from the UWM homepage under the *Current Students* dropdown. The assignments are in the first module "Homework for the First Day". The surveys are designed to get some general information about you and your attitudes towards mathematics. They are designed to help your instructor better tailor the course to fit you. Please take the time to fill them out, they are counted as an assignment.

When you come to your first class, please make sure you have completed all the surveys, the syllabus quiz and the initial knowledge check in ALEKS. These are graded assignments. You will also need to come to class with a binder. In the binder, you should have your syllabus and course outline. You do not need to purchase the workbook from the bookstore as it has changed. You will need to have access to a printer to print the workbook as we move through the course. The entire first chapter is already posted in Canvas in the "Homework for the First Day of Class" module, or you can print them section by section as needed.
Let me know if you cannot access any of the documents. If when you click on a link and it does not work, please cut and paste it in a web browser. You may have to enable your browser to allow you to click the links.

You will be using Canvas to link to your ALEKS account. This way most of your ALEKS grades will automatically update in Canvas. You will need to click the McGraw Hill Campus link and follow the instructions in Canvas. Please use the financial aid code XXXXX-XXXXX-XXXXX-XXXXX (or emailed separate) until the first day of class.

Note: Discussion sections will begin the week starting Monday, September 9. Discussion attendance is required the entire semester.

If you have questions make sure to contact me, or come see me the first day of classes. Please note that anyone who does not attend the first class may be dropped from the course. Remember I am here to help you when you need me. **Do not hesitate to get help**. I look forward to an interesting and fun learning experience this semester.

Phone Script

After sending all the syllabus and other attachments - Follow up with telephone call contact with all students about a week or so before classes start (If you cannot get hold of the student via phone send a letter with the same information at their home address –
Read the scripted message or make your own message.

This is XXXX calling from XXXX, Math Department.

Thank you for registering for the Math XXX course.

I am calling to remind you to please check school e-mail.

You will see a message from me containing the syllabus and homework for the first day of classes.

I also wanted to let you know that our course was selected to use Canvas this semester. You can access Canvas just like you would D2L from the XXX homepage. Just click on Canvas instead.

If you have trouble opening these files, please e-mail me.

I look forward to seeing you next week Tuesday.

Let me know if I can help you in any way.

I look forward to having you in my class.

Talk to you soon. Bye!

You can include other items if you choose but at least have the above minimum. Give the students your contact information in case they need to come see you.

Student Information Survey

What name to you prefer to be called?

What is your adviser's name - can be found in PAWS?

What is your adviser's email?

What lecture section are you enrolled in?

[list of the current sections]

What discussion section are you enrolled in?

[list of the current sections]

What is your intended major or career? If you are undecided what type of work are you leaning toward?

When was the last time you took a math class?

- Last school year
- 1-3 years ago
- 4-6 years ago
- more than 6 years ago

What was the last math class you took?

What are your interests or hobbies?

Do you have any learning needs I should be aware of?

Are there things in your life that may impact your studying for this course? (work, care of family, etc.)

Is there anything else you think I should know?

Appendix G – Workbook Section (Addition)

Name: ____

Learning Outcomes:

Students will be able to…

- Identify like terms
- Estimate integer addition
- Add integers
- Add polynomials
- Estimate fraction addition
- Add fractions
- Add rational expressions
- Add radical expressions
- Add complex numbers
- Add units of measurement
- Add numbers written in scientific notation
- Add functions

Video Notes:

Like Terms	
Definition:	Examples:
Non-Examples:	Characteristics:

Addition	
Definition:	Examples:
Non-Examples:	Characteristics:

Visualizing Addition – what makes sense?

Predict the sign of each addition problem using a number line

$34 + 72$

$-347 + 243$

$724 + (-572)$

$-236 + (-729)$

Round each number to the largest place value to estimate the addition

$34 + 72$

$-347 + 243$

$724 + (-572)$

$-236 + (-729)$

Determine the last digit of each addition problem

$34 + 72$

$-347 + 243$

$724 + (-572)$

$-236 + (-729)$

Perform the integer addition

$34 + 72$

$-347 + 243$

$724 + (-572)$

$-236 + (-729)$

Integer Summary

Evaluate	Estimate Sign	Estimate Magnitude	Last Digit	Evaluation	Correct?
$34 + 72$					
$-347 + 243$					
$724 + (-572)$					
$-236 + (-729)$					

Visualizing Integer Addition – Method 2

$5 + 3$

$5 + (-3)$

$-5 + 3$

$-5 + (-3)$

Adding polynomials

$(4b^3 + 7b^2 + 3b) + (5b^3 + 4b^2 + 2b + 5)$

$(4x^3y + 5x^2y^2 + 7) + (7x^2y^2 + 4xy^2 + 10)$

$7x^2 + (-8x) + 3 + (-6x^2) + 2x + (-4)$

Estimate fraction addition – using the number line to predict the sign

$\frac{4}{5} + \frac{3}{5}$

$\frac{1}{4} + \frac{3}{5}$

$-\frac{2}{3} + \frac{5}{6}$

Draw a model to show the fraction addition and then write each numerically

$\frac{4}{5} + \frac{3}{5}$

$\frac{1}{4} + \frac{3}{5}$

$-\frac{2}{3} + \frac{5}{6}$

Evaluate	Estimate Sign	Estimate Magnitude	Written Numerically	Evaluation	Correct?
$\frac{4}{5} + \frac{3}{5}$					
$\frac{1}{4} + \frac{3}{5}$					
$-\frac{2}{3} + \frac{5}{6}$					

Simplify the following rational expressions

$\frac{x}{7} + \frac{4x}{21}$

$\frac{9}{x} + \left(-\frac{15}{x}\right)$

$\frac{9}{x} + \left(-\frac{8}{5}\right)$

Simplify the following radical expressions

$3\sqrt{7} + \left(-9\sqrt{7}\right)$

$7\sqrt[3]{x} + 3\sqrt[4]{y} + 13\sqrt[3]{x} + \left(-19\sqrt[4]{y}\right)$

Simplify the following

$(7.1 + 2.4i) + (-9.3 + 7.9i)$

$9cm^2 + 8cm + 15cm + 32cm^2$

$2.4 \times 10^{14} + 9.7 \times 10^{14}$

Add the functions

Let $f(x) = 3x + 7 \; and \; g(x) = 7x + 5$

Find:

$(f + g)(x)$

$(f + g)(8)$

$(f + g)(\frac{2}{5})$

Exercises:

Learning Outcome: Identify like terms

1. Answer true or false. Justify your answer.
   a. $3cm + 7cm = 10$
   b. $4cm + 5cm^2 = 9cm^3$
   c. $10in + 8in = 18in$
   d. $\frac{4}{x} + \frac{8}{x} = \frac{12}{x}$
   e. $4x + 5y = 9xy$
   f. $7xy^2 + 5xy^2 = 12xy^2$
   g. $\sqrt[3]{7+x} + 3\sqrt[3]{7+x} = 4\sqrt[3]{7+x}$
   h. $3g + 3kg = 6kg^2$
   i. $3.24 \times 10^4 + 2.3 \times 10^5 = 5.54 \times 10^9$
   j. $7.5 \times 10^4 + 2.1 \times 10^4 = 9.6 \times 10^4$

Learning Outcome: Estimate integer addition

1. Using the number line provided, predict the sign of each of the addition problems. Do not evaluate first.
   a. $56 + 77$
   b. $237 + (-132)$
   c. $(-3425) + 73 + 356$
   d. $0.523 + 2.34 + (-0.732)$

e. $237 + 468$

2. Estimate each of the following addition problems by first rounding to the largest place value and then performing the addition. Do not evaluate first.
   a. $56 + 77$

   b. $237 + (-132)$

   c. $(-3425) + 73 + 356$

   d. $0.523 + 2.34 + (-0.732)$

   e. $237 + 468$

3. Determine the last digit of each of the addition problems. Do not evaluate the entire problem.
   a. $56 + 77$

   b. $237 + (-132)$

   c. $(-3425) + 73 + 356$

   d. $0.523 + 2.34 + (-0.732)$

   e. $237 + 468$

Learning Outcome: Add integers

4. Using an algebra tile diagram or + and – signs, evaluate each of the following integer addition problems.
    a. $5 + 4$
    b. $-5 + 4$
    c. $5 + (-4)$
    d. $-5 + (-4)$
5. Evaluate each of the following expressions.
    a. $56 + 77$
    b. $237 + (-132)$
    c. $(-3425) + 73 + 356$
    d. $0.523 + 2.34 + (-0.732)$
    e. $237 + 468$

Learning Outcome: Add polynomials

6. Simplify the following by combining like terms.
   a. $-3y^2 + (-2y) + 7 + (-5y^2) + 1$
   b. $u^2 + (-5u) + 6 + 2u^2 + 3u + 4$
   c. $(5x^2y^7 + 15xy^2 + 7) + (5xy^2 + (-5x^2y^7) + 4)$
   d. $7a^2 + 4b^3 + (-7a^2) + (-4b^2)$

Learning Outcome: Estimate fraction addition

7. Using the number line provided, predict the sign of the fraction addition problem. Do not evaluate first.
   a. $\frac{3}{5} + \frac{7}{5}$
   b. $\frac{3}{4} + \frac{2}{7}$
   c. $-\frac{7}{10} + \frac{13}{15}$
   d. $\frac{7}{3} + \left(-\frac{17}{6}\right)$

8. Estimate each of the following fraction addition problems by rounding to the nearest benchmark fraction and then addition. Do not evaluate first.
   a. $\frac{3}{5}+\frac{7}{5}$

   b. $\frac{3}{4}+\frac{2}{7}$

   c. $-\frac{7}{10}+\frac{13}{15}$

   d. $\frac{7}{3}+\left(-\frac{17}{6}\right)$

Learning Outcome: Add fractions

9. Using the figures provided, first model each of the fractions. Then, find an equivalent fraction with a common denominator. Next, add the fractions. Last, write the addition statement numerically.
   a. $\frac{3}{4}+\frac{2}{5}$

   Answer:

   Written Numerically:

   b. $\frac{2}{3}+(-\frac{3}{5})$

   Answer:

   Written Numerically:

c. $-\frac{2}{3}+\frac{1}{2}$

Answer:

Written Numerically:

10. Evaluate each of the following expressions
    a. $\frac{3}{5}+\frac{7}{5}$

    b. $\frac{3}{4}+\frac{2}{7}$

    c. $-\frac{7}{10}+\frac{13}{15}$

    d. $\frac{7}{3}+\left(-\frac{17}{6}\right)$

Learning Outcome: Add rational expressions

11. Simplify the following rational expressions.
    a. $\frac{4}{x}+\frac{7}{9}$

    b. $-\frac{3}{a}+\frac{2}{a}$

    c. $\frac{x}{4}+\left(-\frac{3}{5}\right)$

    d. $\frac{y}{8}+\frac{y}{12}$

Learning Outcome: Add radical expressions

12. Simplify the following radical expressions
    a. $\sqrt{6}+5\sqrt{6}+\left(-7\sqrt{6}\right)$
    b. $4\sqrt{7x}+7\sqrt{x}+3\sqrt[3]{7x}+\sqrt[3]{x}+\sqrt{7x}+5\sqrt{x}$
    c. $2\sqrt{3}+\sqrt[4]{7}+3\sqrt[3]{5}+4\sqrt{3}+9\sqrt[3]{5}$
    d. $3.5\sqrt{x}+4.3\sqrt{y}+\left(-7.1\sqrt{x}\right)+\left(-2.1\sqrt{y}\right)$

Learning Outcome: Add complex numbers

13. Add the following complex numbers
    a. $(12+7i)+(4+8i)$
    b. $5i+(9+2i)$
    c. $(7+3i)+(9+2i)$
    d. $\left(\frac{3}{4}+\frac{7}{8}i\right)+\left(\frac{2}{3}+\frac{5}{4}i\right)$

Learning Outcome: Add units of measurement

14. Add the following, be sure to include the correct units
    a. $5in+13in+7in$
    b. $6cm^2+15cm^3+7cm+21cm^3$
    c. $8mi+56ft+45mi$
    d. $\frac{3}{4}c+\frac{2}{3}c+\frac{7}{8}c$

Learning Outcome: Add number written in scientific notation

15. Add the following numbers written in scientific notation. Your final answer should be written in scientific notation
    a. $3.247 \times 10^7 + 4.89 \times 10^7$
    b. $8.23 \times 10^{-5} + (-3.24 \times 10^{-5})$
    c. $7.235 \times 10^{15} + 5.794 \times 10^{15}$
    d. $9.25 \times 10^{-14} + 6.32 \times 10^{-14}$

Learning Outcome: Add functions

16. If each of the functions are defined as follows, add the functions as described.
    $f(x) = 7x + 3,\quad g(x) = 9x + (-7),\quad h(x) = 3x^2 + 2x + 9,\quad k(x) = 5x^2 + 7x + 8$
    a. $(f + g)(x)$
    b. $(g + (-f))(x)$
    c. $(h + k)(x)$
    d. $(g + k)(x)$
    e. $(f + g)(7)$
    f. $(h + k)(-4)$
    g. $(g + k)(2)$
    h. $(g + (-f))(\frac{4}{3})$

Applications:

17. Based on your answers to questions 2-4, determine if your answer in 5 is reasonable. Explain your reasoning.
    a. $56 + 77$
    b. $237 + (-132)$

c. $(-3425) + 73 + 356$

d. $0.523 + 2.34 + (-0.732)$

e. $237 + 468$

18. Based on your answers to questions 7 and 8, determine if your answer in 10 is reasonable. Explain your reasoning.
    a. $\frac{3}{5} + \frac{7}{5}$

    b. $\frac{3}{4} + \frac{2}{7}$

    c. $-\frac{7}{10} + \frac{13}{15}$

    d. $\frac{7}{3} + \left(-\frac{17}{6}\right)$

19. Using the concept of like terms, simplify the following expressions. Do Not distribution the coefficient first.
    a. $7(x-3) + 4(x-3)$
    b. $3x(2x+5) + 7(2x+5)$
    c. $7g(3f+9g) + 12g(3f+9g)$
    d. $15xy\sqrt{4x-3y} + 37xy\sqrt{4x-3y}$
    e. $3x(x-3) + 5(x-3)$
    f. $5y(3y+4) + (3y+4)$

20. Natalie is trying to make salad dressing for her family, but she can't find her measuring cups. She did find a mixing container that has measurements on the side, that she can mix everything in.
   a. She found this recipe for Honey Mustard Salad Dressing (https://wholefully.com/healthy-salad-dressing-recipes/)

      ¼ cup Dijon mustard
      ¼ cup honey
      ¼ cup apple cider vinegar
      ¼ cup extra virgin olive oil
      Salt and pepper

      She only wants to use her mixing container to do the measurements. How far does she have to measure after each addition to know she has the correct amount added?

Ingredient	Measurement
Dijon Mustard	¼ c
Honey	
Apple cider vinegar	
Extra virgin olive oil	

   b. Natalie realized she didn't have honey, so she looked for a different recipe. She found a recipe for Balsamic Vinaigrette Salad Dressing at the same website.

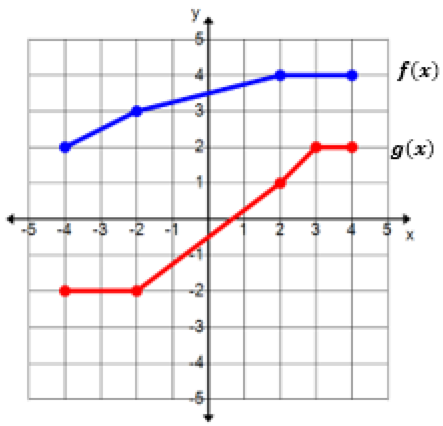
      $\frac{1}{2}$ cup extra virgin olive oil
      $\frac{2}{3}$ cup balsamic vinegar
      $\frac{1}{8}$ cup Dijon mustard
      $\frac{1}{8}$ cup sugar
      1 clove garlic
      Salt and pepper.

      How far does she have to measure after each addition to know she has the correct amount added?

Ingredient	Measurement
Extra virgin olive oil	½ c
Balsamic vinegar	
Dijon Mustard	
Sugar	

21. Ellen decided to keep track of how much she spends in gas each month. She filled up four times in the month of September and spent \$35.43, \$27.98, \$29.74 and \$32. 56. How much did she spend total for gas in September?

22. Find the values from the graph:



a. $g(3)$

b. $(f+g)(-4)$

c. $(g+f)(2)$

d. $(f+g)(-2)$

e. $f(4)+5$

f. $(g+f)(-2)+(-7)$

Discussion Questions:

23. In Exercise 4 all the addition problems looked similar. Explain the how to determine the sign based on zero pairs (not on a rule)

24. In your group, plan a trip from Milwaukee, WI to San Diego, CA. You will need to take several days to complete the trip. You should plan on traveling at least 5 days. Determine where you are going to stop each day, approximately how many hours you travel each day and approximately how many miles you traveled each day. (You do not need to use all the rows provided)

Day	Starting from	Ending at	Time	Miles
1	Milwaukee			
2				
3				
4				
5				
6				
7				
8				
9				
10				
Total				

Google says the distance from Milwaukee, WI to San Diego, CA is 30 hours and 2,117 miles. How does your trip compare to this?

25. Explain how adding 253 + 382 is similar to combining $(2x^2 + 5x + 3) + (3x^2 + 8x + 2)$.

26. Explain the similarities of adding the different types of objects we did in this section.

Review:

Learning Outcome: Determine the domain and range of relations, Determine the input and output of a function

27. Given the following relation, answer the questions

x	$f(x)$
3	7
5	10
7	15
9	23
11	27
13	35

a. Is this a function? Why or why not?

b. What is the domain of the relation?

c. What is the range of the relation?

d. If it is a function, find the following values:

i. $f(5)$

ii. $f(9)$

iii. $f(10)$

iv. For what value(s) of x is $f(x) =$ 35?

v. For what value(s) of x is $f(x) =$ 7?

vi. For what value(s) of x is $f(x) = 0$?

Wrap Up:

I can…

	I need help	I'm getting there	I'm almost there	I understand
Identify like terms				
Estimate integer addition				
Add integers				
Add polynomials				
Estimate fraction addition				
Add fractions				
Add rational expressions				
Add radical expressions				
Add complex numbers				
Add units of measurement				
Add numbers written in scientific notation				
Add functions				

Additional Notes:

Appendix H – Conceptual Learning In-class Activities

Activity: Finding Fractions/Decimal Equivalents using a Number Line

Name ______________________________

1. Use the different colored strips to construct each of the following.
2. Fold one strip in half and mark the fold with $\frac{1}{2}$
3. Fold one strip in quarters and mark each fold with $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ respectively.
4. Fold one strip in eigths and mark each of the fractions.
5. Fold one strip in thirds and mark each of the fractions.
6. Fold one strip in fifths and mark each of the fractions.
7. Fold one strip in tenths and mark each of the fractions.

What is the increment of the number line you were given? Write as a fraction and a decimal number.

Line up each of your colored stips and your number line. Fill in the table to the right with equivalent statements.

Fraction(s)	**Decimal**
	0.1
	0.125
$\frac{1}{5}$,	
$\frac{1}{4}$,	
$\frac{3}{10}$	
	$0.\overline{3}$
	0.375
	0.4
$\frac{1}{2}, \frac{2}{4}$,	
$\frac{3}{5}$,	
	0.625
	$0.\overline{6}$
$\frac{7}{10}$	
$\frac{3}{4}$,	
	0.8
	0.875
$\frac{9}{10}$	

Name: ___________________________

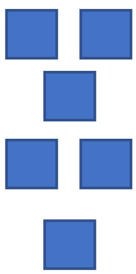
Introduction to Algebra Tiles

Algebra tiles will help you visually see the concepts and principles used in Algebra.

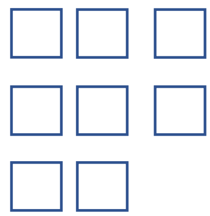
Each small square tile represents the value of 1. The red (grey) side of the tile represents the value of -1

1. Use the small square tiles to represent each number below. Make a sketch of your model.
    a. 7
    b. -5
    c. 4 degrees below zero
    d. 8 feet above sea level

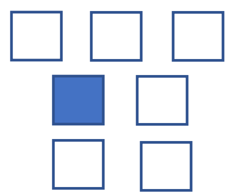
2. What number does each model represent?
    a.



b.

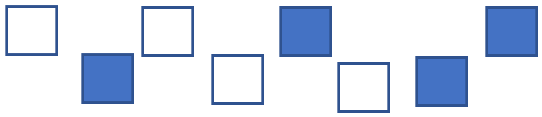


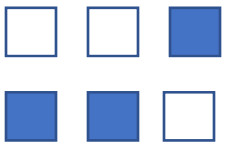
c.

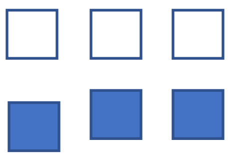


One of the basic rules in algebra is "You can only add zero" to an equation or an expression.

Look at this visually with algebra tiles.

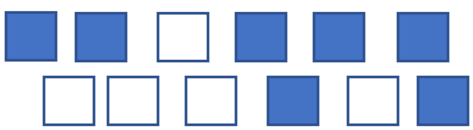
3. Write an equation for each diagram.
   a.


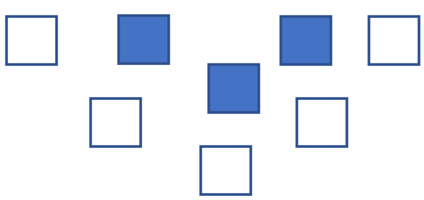
   b.


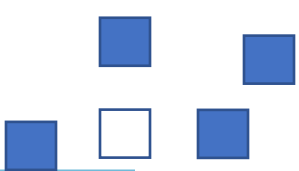
   c.


4. Draw two different models that represent zero.
   a.
   b.

In the previous two problems you saw and represented zero pairs. Using zero pairs you can model many different values.

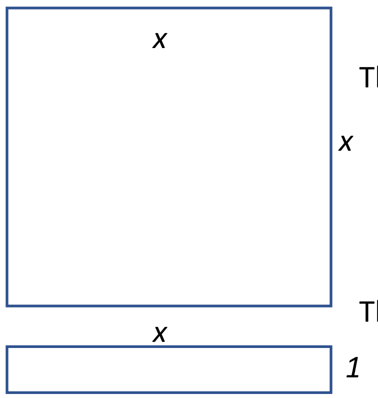
5. Remove (cross out) the zero pairs and determine the value of each integer modeled.
   a.


   b.


   c.


6. Using zero pairs (you must use both color tiles) model these integers.
    a. 3
    b. -7
    c. -3
    d. 5

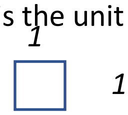
We will use the value of *x* to represent the length of the larger tiles. Then we can name each of the Algebra tiles using the area of each as follows:



This is the x^2 tile because the area of the tile is $x*x=x^2$

This is the x tile because the area of the tile is $1*x=x$

This is the unit tile because the area of the tile is 1*1=1

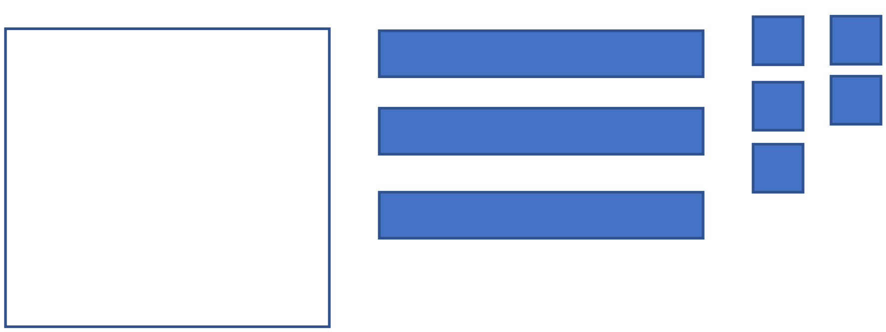
We 

can use these tiles together to represent quadratic algebra expressions.

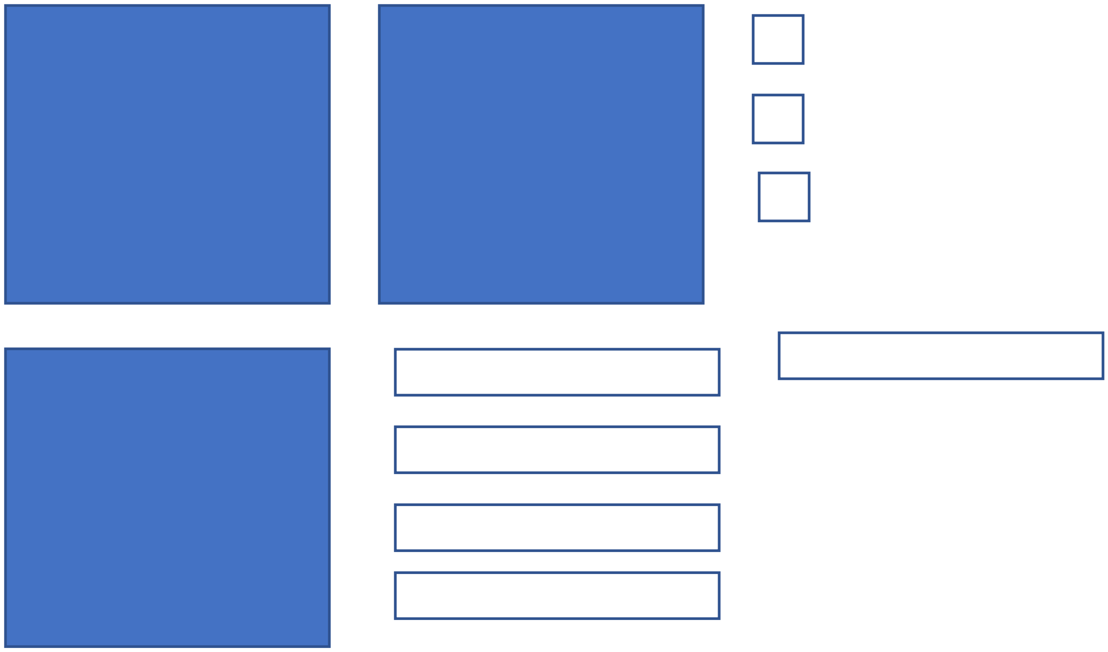
7. Use Algebra Tiles to represent the following polynomials
    a. $x^2 + 3x + 4$
    b. $5x + 2$
    c. $3x^2 + 7x$
    d. $2x^2 + 3$

If you use the red side of the tile you can model negative values.

8. Write the polynomial represented by the following models.
    a.



b.



Above we named the large length *x* and the small length 1. We can name the length other names if we are representing different polynomials.

9. State what you would name the tiles and then represent the following polynomials:
   a. $3c^2 + 4cd - 2d^2$

Large Square _______________

Rectangle __________________

Small Square _______________

   b. $3x^2 - 4xy - 2y^2$

Large Square _______________

Rectangle __________________

Small Square _______________

   c. $7 + 2y + 3y^2$

Large Square _______________

Rectangle __________________

Small Square _______________

Appendix I – Scoring Rubric

Diagram Rational Numbers

Model

Question 1 (diagram column), 10a, b, e

Score	Description
4	Students demonstrate the ability to diagram all fractions using a model. The correct number of divisions are indicated by the denominator and the correct number of partitions are shaded based on the numerator. The improper fractions model shows the correct number of whole objects and the whole was divided into equal parts and completely shaded. All partitions are approximately equal in size.
3	All models are diagramed. There is one minor error partitioning either one denominator or one numerator. There may be one less or one extra partitioning or shading resulting from a miss-count on one fraction. All partitions are approximately equal in size.
2	All models are shown, and the proper fractions are modeled correctly. Improper fractions are not modeled as a whole with an extra part. Or, fractions are modeled with the correct number of partitions and shaded parts, but the partitions are not approximately equal. (For example, modeling ⅓ as 
1	The models show that students know fractions are part of a whole, but incorrect partitions are shown, and the incorrect number of partitions are shaded.
0	No understanding or skill demonstrated.

Number Line/Ruler

Question 1 (number line column), 3a, b, d 4

Score	Description
4	Correctly diagram given rational numbers using the number line. The correct number of partitions are indicated by the denominator and the correct number of partitions are shaded or marked based on the numerator. The fraction number lines are partitioned into units and then partitioned into equal increments based on the denominator of the fraction. The integer number line is partitioned into equal increments of 1. The negative numbers are to the left of zero and the positive numbers are to the right of zero. All partitions are approximately equal in size.
3	Correctly diagram given rational numbers using the number line. The number lines are partitioned into equal increments of the number line as a whole. The distance of 3 is not used. The integer number line has equal partitions but does not use increments of 1. All values are placed on the number line in the correct position.
2	Equal increments are not used on the number line, but values are placed approximately correctly relative to other values.
1	Equal increments are not used, and the values are not placed approximately correctly relative to other values.
0	No understanding or skill demonstrated.

Distance

Question 3c, 4a, b

Score	Description
4	The distance is correct. The placement of the distance on the ruler is on both sides of the indicate value.
3	The distance is correct. The placement of the distance on the ruler is only on one side of the indicated value.
2	The numerical value of the distance is correct, but given as a negative value. The students demonstrate finding the difference of the two numbers but performed the subtraction incorrectly.
1	The student did a different operation other than subtraction. The student attempted to find the distance but did not perform the correct operation.
0	No understanding or skill demonstrated.

Decimal Representation

Question 1 (decimal column)

Score	Description
4	The decimal representation is correct. The repeating decimals are listed as repeating decimals.
3	The decimal representation is correct with rounding. The repeating decimals are not listed as repeating.
2	The proper fractions have the correct decimal representation. The improper fractions are not represented as values greater than one.
1	The values are listed as decimal numbers, but they are not correct.
0	No understanding or skill demonstrated.

Equivalent Fractions

Question 2

Score	Description
4	All equivalent fractions are correctly indicated. All justifications are correct with mathematically correct written work.
3	All equivalent fractions are correctly indicated. Some justifications have a computation error or written work has one minor mathematical error.
2	Some equivalent fractions are not correctly identified, but responses match the the justification given. The justification demonstrates misconceptions in fraction equivalence.
1	Equivalent fractions are not correctly identified or responses do not match the justification given.
0	No understanding or skill demonstrated.

Ordering Fractions

Question 5

Score	Description
4	All numbers are ordered least to greatest. All justification exhibits mathematically correct reasoning.
3	Numbers are ordered least to greatest or greatest to least. Justification lacks solid mathematical reasoning. (For example, stating that the greatest number is the greatest without reasoning.)
2	Numbers are ordered least to greatest or greatest to least with no more than one error. The whole number is misplaced or signs are not considered. Justification is not mathematically correct or missing.
1	Numbers are not ordered correctly. Justification is not mathematically correct or missing.
0	No understanding or skill demonstrated

Adding/Subtracting Fractions

Question 4a, 6a, b, e, 10d

Score	Description
4	All computations are complete and correct. Answers are written in lowest terms as a single fraction.
3	All answers are complete and correct. Answers are written as a single fraction, but not in lowest terms or as a mixed number. Finding an equivalent fraction is written as multiplying/dividing by a single number not one. (For example, $\frac{3}{5} * 2 = \frac{6}{10}$.) Students have a minor numerical calculation error (For example 2 + 4 = 7)
2	Students find an incorrect common denominator. Subsequent work is correct based on error.
1	The incorrect operation is used. Student perform the operation across numerators and denominators without attempting to find a common denominator. The student attempt to use cross-multiplication, cross-addition, or cross-subtraction.
0	No understanding or skill demonstrated.

Multiplying Fractions

Question 6c, 7a, b, c

Score	**Description**
4	All computations are complete and correct. Answers are written in lowest terms as a single fraction.
3	All answers are complete and correct. Answers are written as a single fraction, but not in lowest terms or as a mixed number. Students have a minor numerical calculation error (For example 2 * 4 = 7)
2	Answers are written as a single fraction, but no computation work is given for Problem 7. The student attempts to find a common denominator.
1	The student attempts to cross multiply. The student attempts to solve for the given variable.
0	No understanding or skill demonstrated.

Dividing Fractions

Question 6d, 7d

Score	Description
4	All computations are complete and correct. Answers are written in lowest terms as a single fraction.
3	All answers are complete and correct. Answers are written as a single fraction, but not in lowest terms or as a mixed number. Students have a minor numerical calculation error.
2	Answers are written as a single fraction, but no computation work is given for Problem 7. The student incorrectly applies a division algorithm. (For example, $\frac{1}{7a} \div 7a = 1$.)
1	The student attempts to cross multiply. The student attempts to solve for the given variable. The student attempts to divide common factors without inverting the second fraction.
0	No understanding or skill demonstrated.

Adding/Subtract Integers

Question 8, 9b, c, e, h

Score	Description
4	All predictions are correct
3	Most predictions are correct. Incorrect responses use multiple negatives (For example, N – (-10) = 6 Students evaluate the problem instead of predicting.
2	Most predictions are incorrect.
1	Student attempted the problem, but multiple responses are missing or all predictions are incorrect
0	No understanding or skill demonstrated.

Multiply/Divide Integers

Question 9 a, d, f, g

Score	Description
4	All predictions are correct
3	Most predictions are correct. Student has one error. Students evaluate the problem instead of predicting.
2	Most predictions are incorrect.
1	Student attempted the problem, but multiple responses are missing or all predictions are incorrect
0	No understanding or skill demonstrated.

Appendix J – Course Syllabus

Math 94 – Spring 2020
Foundations of Elementary Mathematics

Instructor Information

Office Hours: *Tuesday and Wednesday 11 – 12:30 pm and by appointment* – I want you to know that I am available to help you if you are having difficulty. These office hours are when I am guaranteed to be in my office. If they do not work for you, I am on campus many other days and times. Please contact me and we can make arrangements that work for you.

Course Description

Mathematics is alive in the world around us. Understanding and being able to use mathematics helps people make sense of adding numbers all the way to complex systems of equations that help a big business determine their supply chain. Understanding mathematics helps with our daily lives in the grocery store, baking and driving. Everyone can benefit from thinking mathematically. This course is designed to help you gain conceptual understanding of basic mathematics and beginning algebra. The course is designed as a flipped classroom. Therefore, many in-class activities will provide opportunities for you to practice with others, identify your misconceptions and gaps, and as a result enhance your conceptual understanding of mathematics. My role as your instructor is to facilitate your learning by creating in- and out-of-class learning experiences so that you can successfully understand the concepts and procedures explored and practice throughout the course.

The course has a vertical design to the course content. This means that the content is organized in a different order than the traditional sequence in K-12 mathematics courses. This is intended so that you can compare and contrast mathematical ideas, justify your answers, and determine the most efficient method to correctly solve math problems. The course includes practical methods to help you learn how to study mathematics.

Course Outcomes

By the end of the course, you will be able to:

1. Utilize varying information to create and solve mathematically relevant problems
2. Justify mathematical statements
3. Determine the most efficient method to correctly solve basic math and algebraic problems
4. Compare and contrast key mathematical ideas
5. Learn the most effective ways to study for mathematics

Course Materials

- Course workbook – download from Canvas and print yourself
- Purchase an ALEKS code. You can access ALEKS through the McGraw Hill Campus module in Canvas
- Internet access to watch videos, access Canvas, and complete ALEKS assignments
- **2" or 3"** three ring binder and dividers (or two 1" binders)
- About 200 pages of loose-leaf paper
- Pen or pencil and a red pen
- Scientific calculator

How to Succeed

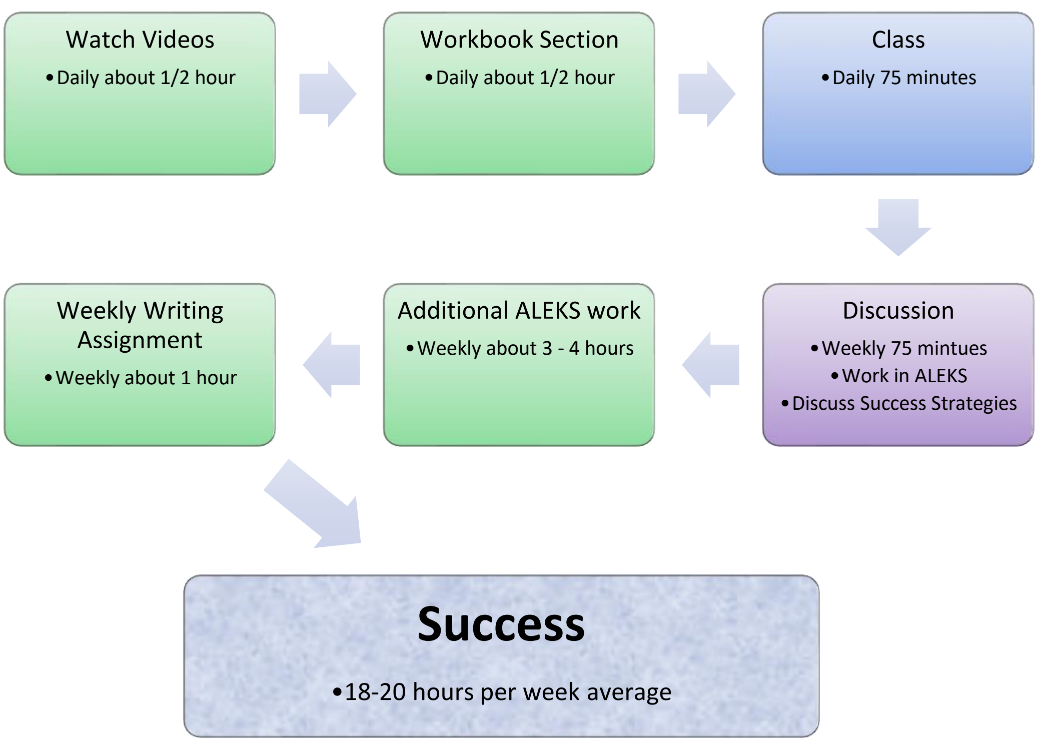
You, the student, are responsible for your own learning. My responsibility as your instructor is to facilitate this learning. This course is set up so that you succeed at learning mathematics and study skills for mathematics. You will learn best by relating what you know (prior learning) to what you are currently learning. By using your prior learning, you will be able to identify your misconceptions and any gaps you may have in your mathematical learning. If you have misconceptions, use the conceptual understanding explored and practiced in class to prove the misconceptions wrong to yourself. This will help you determine the most efficient method to solve any type of math problems. We will be working in pairs, small groups, and whole class to help each other learn – teamwork requires cooperation, it is not competitive or isolated. Thus, to earn credit for in-class activities you must actively participate in your team. You will not earn any credit for just attending class and not participating in the activities.

This course is an accelerated course in a flipped classroom model. Therefore, it is your responsibility to watch the video lectures outside of class so that in class we can work on conceptual understanding in class. To this end your workbook will be checked at the beginning of each class to determine if you have completed the video notes and attempted the workbook exercises. If you make a good attempt at the workbook work, you will earn 1 point per day for a total of 5% of your final grade for the semester. Keeping up with all your work outside of class will help ensure your success in the course. *If you are falling behind or having trouble, please contact your instructor right away.*

You are assigned a separate discussion section that is monitored by a Teaching Assistant (TA). The discussion section provides you with a dedicated time each week learn and practice study skills, complete written work assignments and get help on some problems in your online homework. There is also one proctored assessment during discussion.

Again, if you are having difficulties please contact your instructor. My responsibility as your instructor is to facilitate your learning of mathematics. If you need help, please see me for help.

A typical week will include:



- **This is an average of the time spent for this course. You may need more or less time depending on your understanding of the material.**
- To comply with a Higher Learning Commission requirement, the course syllabus provides information on the *minimum investment* of time required by an average student to achieve the learning goals of the course. Study leading to one semester credit represents an investment of time by the average student of *not fewer than* 48 hours for class contact in lectures, for laboratories, examinations, tutorials and recitations, and for preparation and study; or a demonstration by the student of learning equivalent to that established as the expected product of such a period of study. (The total number of hours should be 48 per credit hour awarded for the course; here, 48 x 6 = 288 hours).
- See https://www4.uwm.edu/secu/docs/faculty/2838_Credit_Hour_Policy.pdf

Course Learning Experiences and Assignments

All aspects of this course are designed to build on one another. Therefore, you as the student, must prepare for each day's lesson and practice problems after each class. I, as your instructor, will provide you with timely and meaningful feedback so you can learn to the best of your abilities. (Green – outside of class learning activities, Blue – in class learning activities, Purple – discussion activities)

Pre- and Post-assessments

(1% – to earn credit for this assignment it must be done before the beginning of the second class):

- ✓ Initial Knowledge Check in ALEKS (access through Canvas)
- ✓ Student Information Survey in Canvas
- ✓ Math Attitude Survey (Pre- and Post-Survey – post-survey due the last day of class) in Canvas

Before class learning experiences

(total of 5% for the semester):

- ✓ Watch videos
- ✓ Come to class with the assigned video notes done and the workbook exercises attempted
- ✓ Rubric for before class learning experience – assessed at the beginning of each class session by your instructor

	Meets Expectations	Does not meet expectations
Workbook work	The workbook section is checked in with the instructor at the **beginning** of class. The assigned video notes are complete Most of the workbook exercises are complete with meaningful work. If there are major gaps, there are questions written where the understanding is not clear	The workbook section is not checked in with the instructor at the beginning of class. The assigned video notes are not complete. The workbook section has major gaps without questions about what is unclear. The workbook section is not brought to class

In-class learning experiences

(excluding exams and retake exam days – total of 15% for the semester):

- ✓ ****In-class learning experiences cannot be made up if you miss class. There may be activities at any time during the class. If you miss an activity you will not earn the credit for that portion of that day.***
- ✓ In-class pair and group activities
- ✓ Formative assessments
- ✓ Class Discussions

Workbook Sections **(total of 5% for the semester)**

- ✓ Students will complete all workbook assignments, work must be complete, correct, and organized. If additional notes are needed, they should be included. Students are allowed one revision. If more revisions are needed the student must meet with the instructor or TA to get help with the mistakes being made.
- ✓ Rubric for Workbook sections

	Meets Expectations	**Not Yet**
Problem work	Almost all problems are complete and correct. One or two problems can have minor errors.	Problems are missing and are incorrect
Corrections (if needed)	All corrections done in a different color. Corrections are complete and correct	Corrections are done in same color as original work. Corrections are incomplete.

- ✓ If you earn a grade of “Not Yet” you are required to make corrections until you earn the grade of “Meets Expectations”

Discussion participation

(total of 5% for the semester):

- ✓ ****Discussion cannot be made up if you miss a discussion***
- ✓ Actively working in ALEKS during the entire discussion period
- ✓ Rubric for Discussion Participation – to be assessed by the TA during each discussion section

	1 point	**½ point**	**0 points**
Timeliness	Arrive on time Ready to work when class starts Leave at the end of the session	Arrive more than 5 minutes late Start work more than 5 minutes late Or Leave more than 5 minutes early	Do not attend Attend less than 30 minutes of the session
Active Participation	Participate in activities in class, and use any extra time constructively	Participate in activities in class, and not use extra time constructively	Do not participate actively in class

ALEKS Objectives **(total of 10% total for the semester– based on the percentage of topics complete for each week's assignment)**

- ✓ Students must complete weekly ALEKS objectives. These are designed to reinforce the learning activities done in the workbook and in class.

Key Assignments *– these are designed to ensure conceptual understanding of mathematics. They will take some time to complete, so do not wait until the last minute. Some may be done as*

- ✓ **Portfolio Review and Exam Analysis – 5% total**
  - Exam 1 Portfolio Review and Exam Analysis – Students will perform a self-assessment of their portfolio and exam. After the self-assessment is complete the student will meet with the instructor to discuss their self-assessment.
  - Exam 2 Portfolio Review and Exam Analysis – Students will perform a self-assessment of their portfolio and exam. Students will turn in a self-assessment rubric and exam corrections
  - Exam 3 Portfolio Review and Exam Analysis - Students will perform a self-assessment of their portfolio and exam. After the self-assessment is complete the student will meet with the instructor to discuss their self-assessment.
- ✓ **Weekly Written Work – 10% total**
- ✓ Students will be required to complete various assignments that contribute toward their success in the course. These assignments will vary on a weekly basis, they may include activities on study skills, or homework assignments in ALEKS that are not part of your pie. These items will be completed in discussion and graded by your discussion teaching assistants.

Exams ***(44% total):***

- ✓ ***Exam 1 – Definitions (6%)***
- ✓ ***Exam 2 – Definitions and Operations (8%)***
- ✓ ***Exam 3 – Solving (10%)***
- ✓ ***Final Exam – Comprehensive (20%)***

- *All exams are cumulative* because all the material builds on itself
- *Exams are not timed*. If you do not finish exams in class, then you must finish them outside class by making an appointment with your instructor.
- ***Exam requirement:*** *Success on your exams depends on you being prepared. Therefore,* you must meet all requirements set below to be eligible to take an exam. Thus, taking an exam is a privilege you earn. Failure to meet these requirements means you will be denied taking in-class exams. Please see the instructor on what to do if you do not meet the requirements below.
  - You must have met the required ALEKS topics target required for the exam.
    - Exam 1 – 175
    - Exam 2 – 280
    - Exam 3 – 400
    - Final Exam - 419
  - You must have completed 100% of any review exams assigned
  - You must have 100% of your workbook assignments turned in and complete (all must be at meets expectations)

- For Exam 2 you must complete an ALEKS Knowledge Check in your discussion section during the assigned week (see below). This Knowledge Checks is not assigned grade points, but you must complete it to be eligible to take Exam 2.

Success in this course depends on you building a solid foundation as we progress through the semester. Therefore, if you score less than 80% on an exam you must retake the exam. This will give you the opportunity to increase your foundational skills and understanding so that you succeed moving forward.

- Exam Retake Requirements:
  - Achieve the above Exam Requirements
  - An Exam Retake may be an oral exam or a written exam.
  - *All retakes will take place one week after the exam is scheduled.*
  - *You must complete an exam analysis and meet with your instructor prior to earning the opportunity to retake the exam.*
  - The maximum score on an exam retake is 80%.

Course Evaluation

Grading Scheme

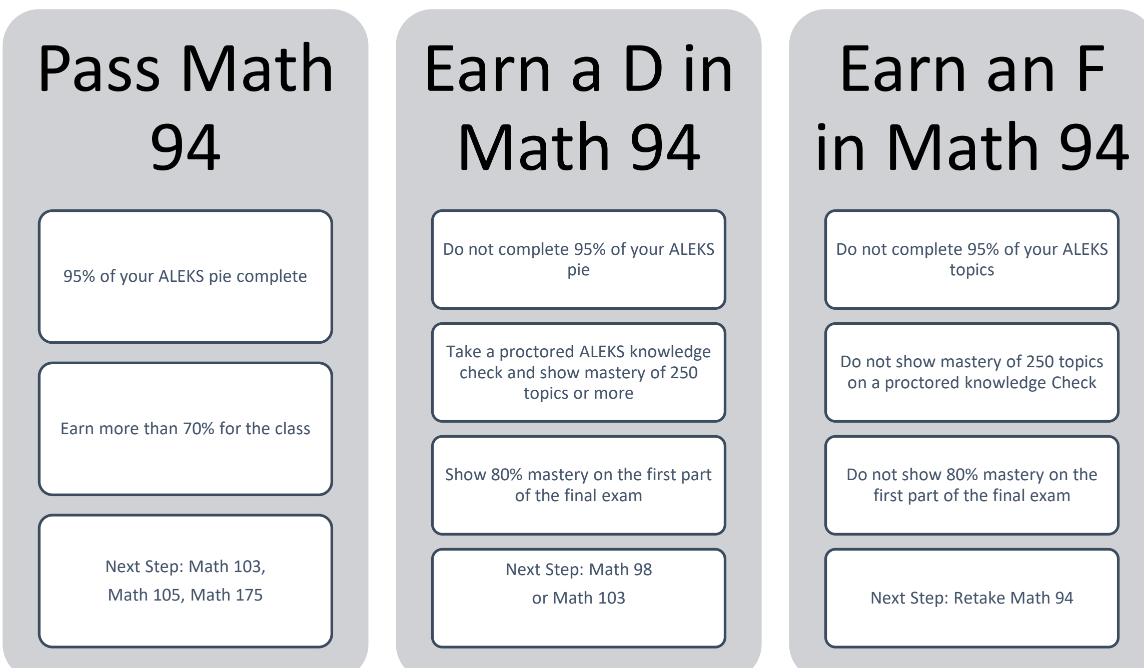
Learning Activity (Green – outside of class learning activities, Blue – in class learning activities, Purple – discussion activities)	**Weight of each Assignment**	**Total points**	**Due Dates**
Pre- and Post-Surveys		1%	2nd day of class
Before Class Learning Experiences		5%	Daily
In-Class Learning Experiences		15%	Daily
Discussion Participation		5%	Weekly
Workbook Sections		5%	Daily
ALEKS objectives		10%	Weekly
Portfolio Review and Exam Analysis • Exam 1 – Self Assessment and Instructor Conference • Exam 2 – Self Assessment • Exam 3 – Self Assessment and Instructor Conference		5%	 Week 7 Week 10 Week 14
Weekly Written Work		10%	Weekly
Exams • Exam 1 – Definition • Exam 2 – Operations • Exam 3 – Solving	 6% 8%	44%	 Week 6 Week 9

• Final Exam – Comprehensive	10% 20%		Week 13 Final
Total		100 %	

To ensure that you are prepared for your next course, you must complete **95% or more** in learning mode (in ALEKS) to earn a C or better in the course. If you do not have more than 95% of the ALEKS topics complete you will earn a C- or lower.

If you complete all the topics in ALEKS learning before the last day of class, your lowest exam score will be replaced with the percentage you earn on the final if the final is greater. For instance, if you finished all your ALEKS topics and scored an 85% on the final, your lowest exam score will be replaced with 85% of the grade assigned to that exam.

- If you have not completed 95% of the topics in ALEKS you will be required to take a proctored comprehensive assessment in ALEKS or show mastery of the basic math material on the comprehensive final. If you do not take a comprehensive assessment in ALEKS or take the comprehensive final you will earn an F for Math 94.
- If you have not mastered more than 250 topics on an ALEKS knowledge check or do not show 80% mastery of the basic math material on the final, you will earn an F for Math 94.
- If you have mastered more than 250 topics or show mastery of the basic math material on the final, you will earn a grade of D. You will then be eligible to take Math 98 or Math 103.



There is one proctored ALEKS knowledge checks scheduled in the semester. This must be taken during your discussion section to be eligible to take Exam 2. If your percent mastery after the knowledge check is more than 15% lower than what you had before the knowledge check, you must meet with your instructor.

Your instructor reserves the right to assign a proctored ALEKS assessment at any point in the semester for any reason.

FINAL GRADES

Final grades will be determined on the following scale if 95% of your ALEKS pie has been completed.

A – 90%-100%

B – 80%-89%

C – 70-79%

D – 60-69% (or your ALEKS pie is not complete, but you have mastered more than 250 ALEKS topics – see above)

CALCULATORS

Calculators are used in everyday life. Your job in this course is to prove that you know what comes out of your calculator is correct by being able to estimate and justify your answers. To that end, you will be allowed a scientific calculator only for some in-class experiences. A calculator will be allowed on exams after you show that you know approximate what the answer should be.

No cell phones, smart watches etc. will be allowed for use as a calculator.

ATTENDANCE

One of the major contributors to your success in this course is attending class regularly. You are expected to attend all class periods. If you must miss a class for any reason, please inform your instructor as soon as possible so you can learn what material you have missed. If you miss a class for any reason, you are not allowed to make up the work for in-class learning experiences. If you miss more than 8 class periods (two weeks of class), the **highest** grade you can earn is a C.

CLASSROOM ETIQUETTE

Help yourself and help your classmates by observing good classroom etiquette:

- Avoid side conversations
- Pair and small group activity etiquette
- Make sure to treat all group members with respect
- Everyone will struggle in this course at some point. It is ok to struggle with your learning.
- Make sure to ask for help from your group mates first, then ask the instructor. Learning different methods from others will help you gain a better understanding.
- It is not ok to yell at the instructor or other classmates when you are struggling
- It is not ok to dismiss another student that is struggling. You will need help at some point in the semester, make sure to help others when you understand the material
- Cell phones are a distraction for others in the class. Put your phone in your backpack until class time is over.

IMPORTANT DATES

September 3	First day of classes **(No discussion this week)**
September 9	Math department's last day to add a class
September 16	Last day to add, last day to change to or from credit/no credit/audit status
September 30	Last day to drop full-term classes without notation of "W" on academic record
October 10	**Exam 1**
October 14-18	*Portfolio Review and Exam Analysis – Exam 1*
October 14-18	**Proctored ALEKS Knowledge Check**
October 31	**Exam 2**
November 4-8	*Portfolio Review and Exam Analysis – Exam 2*

November 10 Last day to drop or withdraw from full-term classes

November 27-December 1Thanksgiving Break (**No classes)**

November 26 Exam 3

December 2-6 Portfolio Review and Exam Analysis – Exam 3

December 9-12 Last week of classes **(Discussion is extra credit this week)**

December 12 Last day of classes

December 13 Study Day

Monday, December 16 7:30 – 9:30 am **Comprehensive Final Exam (paper/pencil)**

Curriculum Vitae

Leah Rineck

(262)305-9440

lmrineck@uwm.edu

Education

PhD in Urban Education – Mathematics Education Anticipated August 2020

University of Wisconsin – Milwaukee

MS in Mathematics August 2006

University of Wisconsin – Milwaukee

BS in Mathematics December 1998

University of Wisconsin – Eau Claire

Teaching Philosophy

Integrating new approaches in developmental mathematics to better serve students that are underprepared.

Teaching Experience

Department of Educational Psychology, University of Wisconsin-Milwaukee Fall 2019-present

Instructor, Educational Statistical Methods I Fall 2019

This is a large lecture course that focuses on descriptive and inferential statistics.

- Developed course material
- Graded student's exams and project
- Assisted the teaching assistant with grading questions

Department of Mathematical Sciences, University of Wisconsin – Milwaukee Fall 2006-present

Coordinator and Instructor, Preparation for College Mathematics *Fall 2014 – present*

This is a flipped classroom, modular designed course. It was fully rolled out to 16 sections fall 2014. The course ranges from basic math to beginning/intermediate algebra. The class meets four days a week for 75 minutes a day. There is a mandatory 75 minute discussion every week where students are required to work on their online homework.

- Develop all class materials, including lesson plans, daily worksheets, exams, and syllabus
- Mentor up to 10 instructors per semester
- Coordinate discussion teaching assistants
- Handle any student complaints, or difficulties
- Hold bi-weekly meeting with instructors to prepare for the upcoming material
- Invite campus resources to the bi-weekly meetings to make instructors aware of the resources on campus

Interim Coordinator and Instructor, Preparation for College Mathematics/Beginning Algebra – Intermediate Algebra Combined *Fall 2006 – Spring 2014*

This was a modified emporium model classroom. Space was limited to 25 students. Students were given one on one attention. The class met twice a day five days a week, 75 minutes in the morning and 50 minutes in the afternoon with two different instructors. The class used an adaptive software program as the main form of instruction.

- Coordinate with the afternoon instructor regarding grading and students
- Make sure the students were on pace to finish the course
- Graded all assessments

Instructor, Intermediate Algebra *Spring 2014*

This was a pilot course in which there was a short lecture on new material every day and then students were given a worksheet to work on together. All sections of the course are now being taught this way.

- Helped develop worksheets
- Helped develop lecture notes
- Graded all students work

Instructor, Contemporary Application of Mathematics *Spring 2008*

- Developed all course material
- Graded all student work
- Assisted students with the material

Instructor of record, Mathematics for Elementary School Teachers — *Summer 2006*

- Developed all course material
- Graded all student work
- Assisted students with the material

Discussion Leader, Business Calculus — *Spring 2006*

- Discussed written homework
- Held test prep classes
- Graded all exams

Experience

Digital Faculty Consultant, McGraw Hill

- Have site visits to campuses
- Consultant via phone
- Discuss best practices using ALEKS
- Discuss best practices for a flipped classroom

Honors/Awards/Scholarships

Amy Tessmer Boening Scholarship	Fall 2019
Amy Tessmer Boening Scholarship	Fall 2018
UWM Outstanding Teaching Award	Fall 2017
Amy Tessmer Boening Scholarship	Fall 2017
ARC Excellence Award from UW-Milwaukee Accessibility Resource Center	Spring 2016

Interviews

Morello, Rachel, "Milwaukee Teachers Weigh In: What's the Formula to Success in Math?" Milwaukee Public Radio, Feb 6, 2017, http://wuwm.com/post/milwaukee-teachers-weigh-whats-formula-success-math

Publications

John R. Reisel, University of Wisconsin, Milwaukee; Marissa Jablonski, University of Wisconsin, Milwaukee; Leah Rineck; Ethan V. Munson, University of Wisconsin, Milwaukee; Hossein Hosseini, University of Wisconsin, Milwaukee, "**Analysis of Math Course Placement Improvement and Sustainability Achieved Through a Summer Bridge Program**", 2012 ASEE Annual Conference

John R. Reisel, University of Wisconsin - Milwaukee; Leah Rineck; Marissa Jablonski,University of Wisconsin, Milwaukee; Ethan V Munson, University of Wisconsin, Milwaukee; Hossein Hosseini, University of Wisconsin, Milwaukee, "**Evaluation of the Impacts of Math Course Placement Improvement Achieved through a Summer Bridge Program**", 2011 ASEE Annual Conference

Marissa Jablonski, University of Wisconsin, Milwaukee; John R. Reisel, University of Wisconsin, Milwaukee; Hossein Hosseini, University of Wisconsin, Milwaukee; Ethan V Munson, University of Wisconsin, Milwaukee; Leah Rineck, **Initial Evaluation of the Impact of Math Study Groups on First-Year Student Course Success**", 2011 ASEE Annual Conference – Poster Session

Informal Publications

Leah Rineck, "Fundamentals of College Mathematics Workbook" 2nd Edition, Informal Publication, August 2019

Leah Rineck, "Fundamentals of College Mathematics Workbook", Informal Publication, August 2016

Leah Rineck and Xianwei VanHarpen, "Math 98/108 Combined Workbook", Informal Publication, August 2015

Leah Rineck, "Flipping a Modular Classroom using ALEKS", Blog Post, February 2015

Leah Rineck, "How ALEKS Helped a Veteran Find Success", Blog Post, February 2015

Presentations

International

Leah Rineck, "Accelerating Students to Credit Bearing Mathematics Classes", ALM 24, 2017

Leah Rineck, "Using Algebra Tiles to Help Integer Understanding", ALM 24, 2017

National

Leah Rineck and Halyley Nathan, “Creating a Course Backwards from the Learning Outcomes”, AMATYC 2019

Leah Rineck, “Number Talks to Increase Student Understanding”, AMATYC 2018

Paul Nolting, Rochelle Beatty, Leah Rineck, “Math and Study Skills Instructors”, NADE 2017

Leah Rineck, “Using Manipulatives to Enhance Learning Basic Concepts”, NADE 2017

Leah Rineck, “Math Success Strategies”, Poster Presentation, NADE 2017

Paul Nolting, Fitzroy Farquherson, Leah Rineck, “Academic Success Press: Integrating Math Study Skills – Classroom, Modular, & Online Approaches”, AMATYC, November 2016

Leah Rineck, Melisa Madsen, “Using a Co-Requisite Model for Integrating Math Study Skills into Developmental Mathematics”, NADE, March 2016

Leah Rineck, “Math Classroom Approaches for the 21st Century – Flipped and Accelerated”, NADE, March 2016

Paul Nolting, Fitzroy Farquherson, Leah Rineck, “Academic Success Press: Integrating Math Study Skills – Classroom, Modular, & Online Approaches”, AMATYC, November 2015

Leah Rineck, Kelly Kohlmetz, “Bridging the Gap from High School to College”, Poster Presentation, AMATYC November 2015

"Flipping and Accelerating Developmental Math", STEM Teaching Strategies Summit, March 2015

Regional

Leah Rineck, "Supporting Developmental Students with Number Talks and Concept Mapping", Wisconsin Mathematics Council, May 2018

Mike Steele, Barbara Bales, Mary Mooney, Jennifer Lawler, Gwyneth Hughes, Mark Schommer, Leah Rineck, and Jenn Kosiak, "College and Career Readiness: Implication for High Schools and Two- and Four-Year Colleges", Wisconsin Mathematics Council, May 2018

Leah Rineck, "Accelerating Developmental Students to Credit Bearing Math Courses", Wisconsin Mathematics Council, May 2016

Workshop on Best Practices using the New Student Interface in ALEKS, CNM, March 2015

Skills/Training

Proficient in the use of Knewton Alta

Proficient in the use of ALEKS

Proficient in the use of MyMath Lab

Completed "How to Learn Math" for Teachers and Parents, Jo Boaler – youcubed.org Summer 2015

Attended Quantway training at Carnegie Summer 2015

Completed "Veterans On Campus" online training Summer 2015

Completed many online training on Flipped Classrooms and Student Engagement

Professional Associations

National Council of Teachers of Mathematics	2016-present
American Education Research Association (AERA)	2016-present
National Association of Developmental Education (NADE) Spring	2013 - present
American Mathematical Association of Two-Year Colleges (AMATYC) Fall	2015 – present

References

Dr. Michael Steele, Professor of Teaching and Learning

2400 E. Hartford Ave., 309 Enderis Hall

University of Wisconsin – Milwaukee

Milwaukee, WI 53211

414.229.4884

steele@uwm.edu

Dr. Kyle Swanson , Dean of the College of Sciences

Founders Hall

Room 205

700 E. Seventh St.

St. Paul, MN 55106-5000

651.793.1681

kyle.swanson@metrostate.edu

Dr. Keven McLeod, Associate Professor of Mathematics

3200 N. Cramer St

University of Wisconsin – Milwaukee

Milwaukee, WI 53211

414.229.5269

kevinm@uwm.edu

Dr. Paul Nolting, Learning Specialist

3547 53rd Ave. W

PMB 132

Bradenton, FL 34210

pnolting@aol.com